### Vehicle Propulsion Systems Lecture 7

Non Electric Hybrid Propulsion Systems

Lars Eriksson Professor

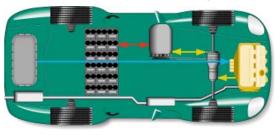
Vehicular Systems Linköping University

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### Hybrid Electrical Vehicles - Parallel

- Two parallel energy paths
- One state in QSS framework, state of charge

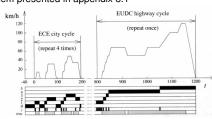


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### Optimization, Optimal Control, Dynamic Programming What gear ratios give the lowest fuel consumption for a given

drivingcycle? -Problem presented in appendix 8.1



Problem characteristics

- ► Countable number of free variables,  $i_{g,j}, j \in [1, 5]$
- A "computable" cost,  $m_f(\cdots)$
- A "computable" set of constraints, model and cycle
- ► The formulated problem min i<sub>g,j</sub>, j∈[1,5]

 $m_f(i_{g,1}, i_{g,2}, i_{g,3}, i_{g,4}, i_{g,5})$ 

s.t. model and cycle is fulfilled

### General problem formulation

Performance index

$$J(u) = \phi(\mathbf{x}(t_b), t_b) + \int_{t_a}^{t_b} L(\mathbf{x}(t), u(t), t) dt$$

System model (constraints)

$$\frac{d}{dt}x = f(x(t), u(t), t), x(t_a) = x_a$$

State and control constraints

$$u(t) \in U(t)$$
  
 $x(t) \in X(t)$ 

### Outline

### Repetition

Short Term Storage

- Hybrid-Inertial Propulsion System
  - Basic princip
  - Medeling
    - Continuously Variable Transmission

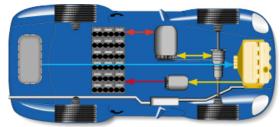
Hybrid-Hydraulic Propulsion System

- Basics
- Wodeli
- Hydraulic Pumps and Motors
- Pneumatic Hybrid Engine Systems

Case studies

### Hybrid Electrical Vehicles - Serial

- Two paths working in parallel
- Decoupled through the battery
- Two states in QSS framework, state of charge & Engine speed



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### **Optimal Control – Problem Motivation**

Car with gas pedal u(t) as control input:

How to drive from A to B on a given time with minimum fuel consumption?

- Infinite dimensional decision variable u(t).
- Cost function  $\int_0^{t_f} \dot{m}_f(t) dt$
- Constraints:

Model of the car (the vehicle motion equation)

$$\begin{array}{lll} m_{v}\frac{d}{dt}v(t) &= F_{t}(v(t), u(t)) & -(F_{a}(v(t)) + F_{r}(v(t)) + F_{g}(x(t))) \\ \frac{d}{dt}x(t) &= v(t) \\ \dot{m}_{t} &= f(v(t), u(t)) \end{array}$$

- Starting point x(0) = A
- End point  $x(t_f) = B$
- Speed limits  $v(t) \le g(x(t))$
- Limited control action  $0 \le u(t) \le 1$

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### Dynamic programming – Problem Formulation

Optimal control problem

$$\min J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t) dt$$
  
s.t.  $\frac{d}{dt}x = f(x(t), u(t), t)$   
 $x(t_a) = x_a$   
 $u(t) \in U(t)$   
 $x(t) \in X(t)$ 

- ► x(t), u(t) functions on  $t \in [t_a, t_b]$
- Search an approximation to the solution by discretizing
   the state space x(t)
  - ► and maybe the control signal *u*(*t*) in both amplitude and time.
- The result is a combinatorial (network) problem

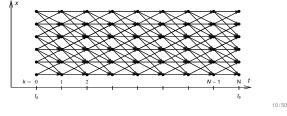
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### Deterministic Dynamic Programming - Basic algorithm

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$
$$x_{k+1} = f_k(x_k, u_k)$$

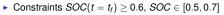
Algorithm idea:

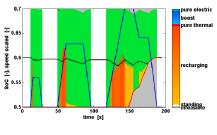
Start at the end and proceed backwards in time to evaluate the optimal cost-to-go and the corresponding control signal.



### Parallel Hybrid Example

- Fuel-optimal torque split factor  $u(SOC, t) = \frac{T_{e-motor}}{T_{aeathor}}$
- ECE cycle





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### Analytical Solutions to Optimal Control Problems

Hamiltonian

 $H(t, q(t), u(t), \mu(t)) = L(t, u(t)) + \mu(t) f(t, q(t), u(t))$ 

Solution (theory from Appendix B)

 $u(t) = \operatorname*{arg\,min}_{u} H(t, q(t), u(t), \mu(t))$ 

with

$$\dot{\mu}(t) = -\frac{\partial}{\partial q} f(t, q(t), u(t))$$
$$\dot{q}(t) = f(t, q(t), u(t))$$

▶ If  $\frac{\partial}{\partial q} f(t, q(t), u(t)) = 0$  the problem becomes simpler  $\mu$  becomes a constant  $\mu_0$ , search for it when solving

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### Outline

Repetition

#### Short Term Storage

Hybrid-Inertial Propulsion Systems Basic principles Design principles Modeling Continuously Variable Transmission

Hybrid-Hydraulic Propulsion Systems Basics Modeling

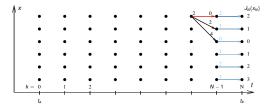
Hydraulic Pumps and Motors

Pneumatic Hybrid Engine Systems

Case studies

# Deterministic Dynamic Programming – Basic Algorithm

#### Graphical illustration of the solution procedure



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### Analytical Solutions to Optimal Control Problems

Core of the problem

$$\min J(u) = \phi(q(t_f), t_f) + \int_0^{t_f} L(t, u(t)) dt$$
  
s.t.  $\dot{q}(t) = f(t, q(t), u(t))$ 

- Hamiltonian from optimal control theory
  - $H(t, q(t), u(t), \mu(t)) = L(t, u(t)) + \mu(t) f(t, q(t), u(t))$

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### ECMS

- $\blacktriangleright$  Given the optimal  $\lambda^*$  (cycle dependent exchange rate between fuel and electricity) .
- Hamiltonian

$$H(t, q(t), u(t), \lambda^*) = P_f(t, u(t)) + \lambda \hat{a}^* P_{ech}(t, u(t))$$

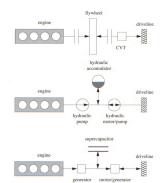
Optimal control action

$$u^*(t) = \operatorname*{arg\,min}_u H(t, q(t), u, \lambda^*)$$

Guess λ<sup>\*</sup>, run one cycle see end SOC, update λ<sup>\*</sup>, and iterate until SOC(t<sub>l</sub>) ≈ SOC(0).

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### Examples of Short Term Storage Systems



### Short Term Storage - F1

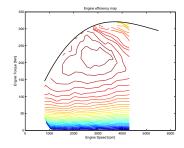
2009 FIA allowed the usage of 60 kW, KERS (Kinetic Energy Recovery System) in F1.

- Technologies:
  - ► Flywheel
  - Super-Caps, Ultra-Caps
  - Batteries

2014, will allow KERS units with 120 kilowatts (160 bhp). -To balance the sport's move from 2.4 I V8 engines to 1.6 I V6 engines.

### **Basic Principles for Hybrid Systems**

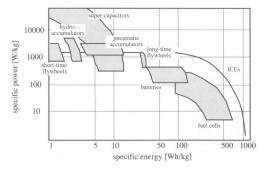
- Kinetic energy recovery
- Use "best" points Duty cycle.
  - Run engine (fuel converter) at its optimal point.
  - Shut-off the engine.



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### Power and Energy Densities

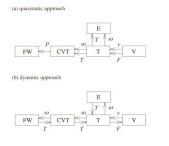
### Asymptotic power and energy density - The Principle



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### Causality for a hybrid-inertial propulsion system



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### Flywheel accumulator - Design principle

Energy stored (SOC):

$$E_f = \frac{1}{2} \Theta_f \omega_f^2$$

Wheel inertia

$$\Theta_f = \rho \, b \, \int_{Area} r^2 \, 2 \, \pi \, r \, dr = \ldots = \frac{\pi}{2} \, \rho \, b \, \frac{d^4}{16} \, (1 - q^4)$$

Wheel Mass

$$m_{f} = \pi \, 
ho \, b \, d^{2} \, (1 - q^{2})$$

Energy to mass ratio

$$\frac{E_f}{m_f} = \frac{d^2}{16}(1+q^2)\omega_f^2 = \frac{u^2}{4}(1+q^2)$$

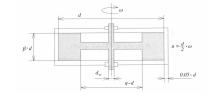
### Outline

### Short Term Storage Hybrid-Inertial Propulsion Systems Basic principles Design principles Modeling Continuously Variable Transmission Hybrid-Hydraulic Propulsion Systems Basics Modeling Hydraulic Pumps and Motors

- Pheumatic Hybrid Engine Syster
- Case studies

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### Flywheel accumulator



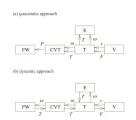
Energy stored (Θ<sub>f</sub> = J<sub>f</sub>):

$$E_f = \frac{1}{2} \Theta_f \omega_f^2$$

Wheel inertia

$$\Theta_f = \rho \, b \, \int_{Area} r^2 \, 2 \, \pi \, r \, dr = \ldots = \frac{\pi}{2} \, \rho \, b \, \frac{d^4}{16} \, (1 - q^4)$$

### Quasistatic Modeling of FW Accumulators

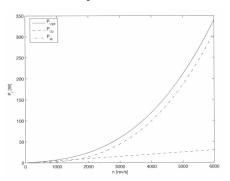


Flywheel speed (SOC)  $P_2(t)$  – power out,  $P_l(t)$  – power loss

$$\Theta_f \omega_2(t) \frac{d}{dt} \omega_2(t) = -P_2(t) - P_l(t)$$

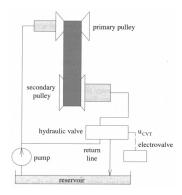
### Power losses as a function of speed

#### Air resistance and bearing losses



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### **CVT** Principle



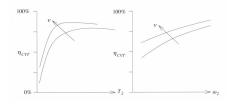
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### **CVT Modeling**

 Transmission (gear) ratio v, speeds and transmitted torques

$$\omega_1(t) = \nu(t) \,\omega_2(t)$$
  
$$T_{t1}(t) = \nu \left(T_{t2}(t) - T_{t}(t)\right)$$

 An alternative to model the losses, is to use an efficiency definition.



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### Outline

Repetition

#### Short Term Storage

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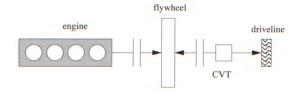
#### Hybrid-Hydraulic Propulsion Systems Basics Modeling

Hydraulic Pumps and Motors

Pneumatic Hybrid Engine System

**Case studies** 

### Continuously Variable Transmission (CVT)



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### **CVT Modeling**

 Transmission (gear) ratio v, speeds and transmitted torques

$$\omega_1(t) = \nu(t) \,\omega_2(t)$$
  
$$T_{t1}(t) = \nu \left(T_{t2}(t) - T_l(t)\right)$$

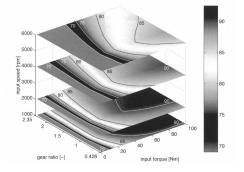
Newtons second law for the two pulleys

$$\Theta_1 \frac{d}{dt} \omega_1(t) = T_1(t) - T_{t1}(t)$$
$$\Theta_2 \frac{d}{dt} \omega_2(t) = T_2(t) - T_{t2}(t)$$

System of equations give

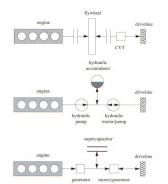
$$T_{1}(t) = T_{l}(t) + \frac{T_{2}(t)}{\nu(t)} + \frac{\Theta_{CVT}(t)}{\nu(t)} \frac{d}{dt} \omega_{2}(t) + \Theta_{1} \frac{d}{dt} \nu(t) \omega_{2}(t)$$
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### Efficiencies for a Push-Belt CVT

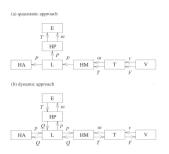


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### Examples of Short Term Storage Systems



### Causality for a hybrid-hydraulic propulsion system



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### Model Simplification

Simplifications made in thermodynamic equations to get a simple state equation.

Assuming steady state conditions.
 –Eliminating θ<sub>g</sub> and the volume change gives

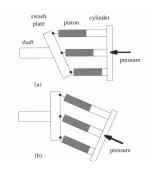
$$p_2(t) = \frac{h A_w \theta_w m_g R_g}{V_g(t) h A_w + m_g R_g Q_2(t)}$$

Combining this with the power output gives

$$Q_2(t) = \frac{V_g(t)}{m_g} \frac{h A_w P_2(t)}{R_g \theta_w h A_w - R_g P_2(t)}$$

- Integrating  $Q_2(t)$  gives  $V_g$  as the state in the model.
- Modeling of the hydraulic systems efficiency, see the book.
   A detail for the assignment
- -This simplification can give problems in the simulation if parameter values are off. (Division by zero.)

### Hydraulic Pumps



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### Modeling of a Hydraulic Accumulator

Modeling principle -Energy balance

$$m_{g}c_{v}\frac{d}{dt}\theta_{g}(t) = -p\frac{d}{dt}V_{g}(t) - hA_{w}(\theta_{g}(t) - \ell$$
-Mass balance  
(=volume for incompressible fluid)  

$$\frac{d}{dt}V_{g}(t) = Q_{2}(t)$$
Power generation  
-Ideal gas law  

$$P_{2}(t) = p_{2}(t)Q_{2}(t)$$

$$p_g(t) = rac{m_g\,R_g\, heta_g(t)}{V_g(t)}$$

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### Outline

#### Repetition

#### Short Term Storage

- Hybrid-Inertial Propulsion Systems Basic principles Design principles Modeling
  - Continuously Variable Transmission

Hybrid-Hydraulic Propulsion Systems Basics Modeling

#### Hydraulic Pumps and Motors

Pneumatic Hybrid Engine Systems

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### Modeling of Hydraulic Motors

Efficiency modeling

$$\begin{split} P_{1}(t) = & \frac{P_{2}(t)}{\eta_{hm}(\omega_{2}(t), T_{2}(t))}, \qquad P_{2}(t) > 0\\ P_{1}(t) = & P_{2}(t) \eta_{hm}(\omega_{2}(t), -|T_{2}|(t)), \qquad P_{2}(t) < 0 \end{split}$$

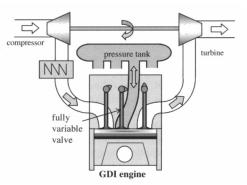
Willans line modeling, describing the loss

$$P_1(t) = \frac{P_2(t) + P_0}{e}$$

 Physical modeling Wilson's approach provided in the book.

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### Pneumatic Hybrid Engine System



### Outline

#### Repetition

Short Term Storage

Hybrid-Inertial Propulsion Systems Basic principles Design principles Modeling Continuously Variable Transmission

Hybrid-Hydraulic Propulsion Systems Basics Modeling

Hydraulic Pumps and Motors

### Pneumatic Hybrid Engine Systems

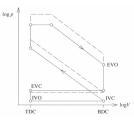
Case studies

### **Conventional SI Engine**

#### Compression and expansion model

 $p(t) = c v(t)^{-\gamma} \qquad \Rightarrow \qquad \log(p(t)) = \log(c) - \gamma \log(v(t))$ 

gives lines in the log-log diagram version of the pV-diagram

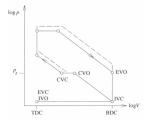


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### Under Charged Mode



### Outline

#### Repetition

#### Short Term Storage

lybrid-Inertial Propulsion Systems Basic principles Design principles Modeling Continuously Variable Transmission

Hybrid-Hydraulic Propulsion Systems

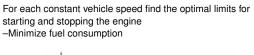
Modeling

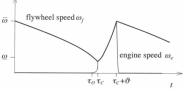
Hydraulic Pumps and Motors

Pneumatic Hybrid Engine Systems

Case studies

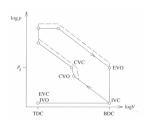
Problem description





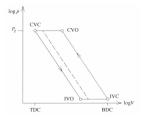
–Solved through parameter optimization  $\Rightarrow$  Map used for control

### Super Charged Mode



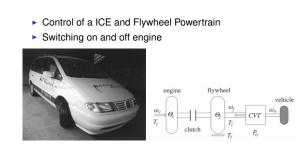
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### Pneumatic Brake System



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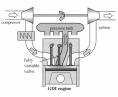
## Case Study 3: ICE and Flywheel Powertrain



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### Case Study 8: Hybrid Pneumatic Engine

- Local optimization of the engine thermodynamic cycle
- Different modes to select between
- Dynamic programming of the mode selection



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