### Vehicle Propulsion Systems Lecture 9

Case Study 6 Fuel Cell Vehicle and Optimal Control

Lars Eriksson Professor

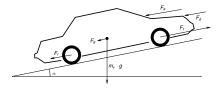
Vehicular Systems Linköping University

May 15, 2017

### The Vehicle Motion Equation

Newtons second law for a vehicle

$$m_{V} \frac{d}{dt} v(t) = F_{t}(t) - (F_{a}(t) + F_{r}(t) + F_{g}(t) + F_{d}(t))$$



- ► F<sub>t</sub> tractive force
- ► F<sub>a</sub> aerodynamic drag force
- ▶ F<sub>r</sub> rolling resistance force
- ▶ F<sub>g</sub> gravitational force
- ▶ F<sub>d</sub> disturbance force

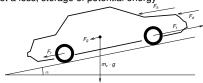
## **Gravitational Force**

Outline

1/43

Repetition

Gravitational load force
 Not a loss, storage of potential energy



▶ Up- and down-hill driving produces forces.

$$F_g = m_v g \sin(\alpha)$$

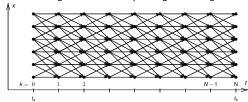
Flat road assumed  $\alpha=$  0 if nothing else is stated (In the book).

Deterministic Dynamic Programming – Basic algorithm

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$
$$x_{k+1} = f_k(x_k, u_k)$$

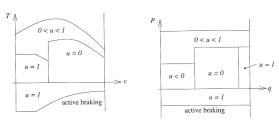
Algorithm idea:

Start at the end and proceed backward in time to evaluate the optimal cost-to-go and the corresponding control signal



On-Line Control – Heuristic Control Approaches

Parallel hybrid vehicle (electric assist)



Determine control output as function of some selected state variables:

vehicle speed, engine speed, state of charge, power demand, motor speed, temperature, vehicle acceleration, torque demand

On-Line Control – ECMS

- ▶ Given the optimal  $\lambda^*$  (cycle dependent exchange rate between fuel and electricity) .
- Hamiltonian

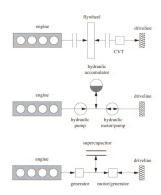
$$H(t, q(t), u(t), \lambda^*) = P_f(t, u(t)) + \lambda^* P_{ech}(t, u(t))$$

Optimal control action

$$u^*(t) = \underset{u}{\operatorname{arg\,min}} H(t, q(t), u, \lambda^*)$$

▶ Guess  $\lambda^*$ , run one cycle see end SOC, update  $\lambda^*$ , and iterate until  $SOC(t_f) \approx SOC(0)$ .

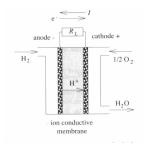
**Examples of Short Term Storage Systems** 

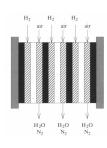


/43

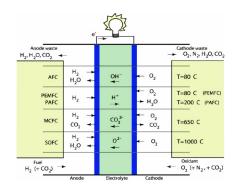
## Fuel Cell Basic Principles

- ► Convert fuel directly to electrical energy
- Let an ion pass from an anode to a cathode
- ► Take out electrical work from the electrons
- Fuel cells are stacked ( $U_{cell} \le 1V$ )





### Overview of Different Fuel Cell Technologies



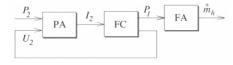
10

# Hydrogen Fuel Storage

- ▶ Hydrogen storage is problematic Challenging task.
- Some examples of different options.
  - High pressure bottles
  - Liquid phase Cryogenic storage, -253°C.
  - Metal hydride
  - Sodium borohydride NaBH4

## Quasistatic Modeling of a Fuel Cell

► Causality diagram



- ► Power amplifier (Current controller)
- ► Fuel amplifier (Fuel controller)
- Standard modeling approach

12/4

#### Fuel Cell Thermodynamics

Starting point reaction equation

$$H_2 + \frac{1}{2} O_2 \Rightarrow 2 H_2 0$$

► Open system energy – Enthalpy H

$$H = U + pV$$

► Reversible energy – Gibbs free energy G

$$G = H + TS$$

Open circuit cell voltages

$$U_{rev} = -\frac{\Delta G}{n_e F},$$

$$U_{id} = -rac{\Delta H}{n_e \, F},$$

$$U_{rev} = \eta_{id} U$$

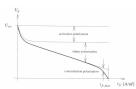
F – Faradays constant ( $F = q N_0$ )

Under load

$$P_{I} = I_{fc}(t) \left( U_{id} - U_{fc}(t) \right)$$

Fuel Cell Performance – Polarization curve

Polarization curve of a fuel cell Relating current density  $i_{fc}(t) = I_{fc}(t)/A_{fc}$ , and cell voltage  $U_{fc}(t)$ 



Curve for one operating condition

- Fundamentally different compared to combustion engine/electrical motor
- Excellent part load behaviorWhen considering only the cell

14/

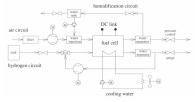
#### Fuel Cell System Modeling

Describe all subsystems with models

$$P_2(t) = P_{st}(t) - P_{aux}(t)$$

$$P_{aux} = P_0 + P_{em}(t) + P_{ahp}(t) + P_{hp}(t) + P_{cl}(t) + P_{cf}(t)$$

em-electric motor, ahp – humidifier pump, hp – hydrogen recirculation pump, cl – coolant pump, cf – cooling fan.



 Submodels for: Hydrogen circuit, air circuit, water circuit, and coolant circuit

#### Outline

Repetition

Case study 6: Fuel Optimal Trajectories of a Racing FCEV Model compilation

Model simplification

Formulating the optimal control problem

**Optimal Controllers Solutions** 

Final Results

Some Additional Material – Fuel Consumption

## **Problem Setup**

▶ Run a fuel cell vehicle optimally on a racetrack



- Start up lap
- Repeated runs on the track
- ▶ Path to the solution
  - ► Measurements Model
  - Simplified model
  - Optimal control solutions

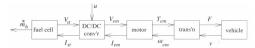
### Problem Setup - Road Slope Given

Road slope  $\gamma = \alpha(x)$ 0.04
0.02
0.02
-0.02
-0.04
0 1000 2000 3000 4000 x [m]

18/4

## **Model Causality**

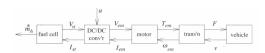
Model causality - Dynamic model



. . . . . .

17/43

## Model Component - Fuel Cell



Current in the cell and losses

$$I_{fc}(t) = I_{fc}(t) + I_{aux}(t)$$

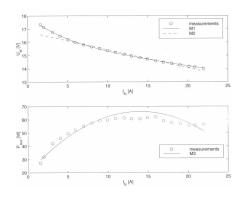
Current and hydrogen flow

$$\dot{m}_H(t) = c_9 I_{fc}(t)$$

▶ Next step: Polarization curve and auxiliary consumption

20/4

#### Fuel Cell - Polarization and Auxiliary Components



21/43

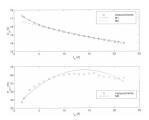
#### Fuel Cell - Polarization and Auxiliary Components

► Polarization curve

$$U_{st}(t) = c_0 + c_1 \cdot e^{-c_2 \cdot I_{fc}(t)} - c_3 \cdot I_{fc}(t)$$

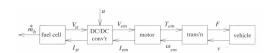
Auxiliary power

$$P_{aux}(t) = c_6 + c_7 \cdot I_{fc}(t) + c_8 \cdot I_{fc}(t)^2$$



22/

#### Model Component - DC Motor Controller



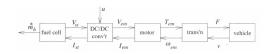
▶ DC motor voltage (from control signal *u*)

$$U_{em}(t) = \kappa \, \omega_{em}(t) + K \, R_{em} \, u(t)$$

Current requirement at the stack

$$I_{st} = \frac{U_{em}(t)I_{em}(t)}{\eta_c U_{st}(t)}$$

#### Model Component - DC Motor



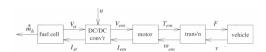
▶ DC motor current

$$I_{em}(t) = rac{U_{em}(t) - \kappa \, \omega_{em}(t)}{R_{em}}$$

▶ DC motor torque

$$T_{em}(t) = \kappa_{em} I_{em}(t)$$

#### Model Component - Transmission and Wheels



▶ Tractive force

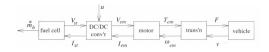
$$F(t) = \eta_t^{\pm 1} \, \frac{\gamma \, T_{em}(t)}{r_{w}}$$

► Rotational speed

$$\omega_{em}(t) = \frac{\gamma v(t)}{r_w}$$

25/43

#### Model Compilation 1 – Vehicle



▶ The vehicle tractive force can now be expressed as

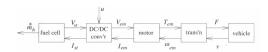
$$F(t) = \frac{\eta_t \, \gamma}{r_w} \, \kappa_{em} \, K \, u(t)$$

► Dynamic vehicle velocity and position model

$$\frac{d}{dt}v(t) = h_1 u(t) - h_2 v^2(t) - g_0 - g_1 \alpha(x(t))$$
$$\frac{d}{dt}x(t) = v(t)$$

26/43

## Model Compilation 2 - Fuel Consumption



Fuel flow,  $\dot{m}_H(t) = c_9 I_{fc}(t)$ 

$$I_{\mathit{fc}}(t) = \frac{P_{\mathit{aux}}(I_{\mathit{st}}(t))}{U_{\mathit{st}}(I_{\mathit{st}}(t))} + \frac{K\,\mathit{u}(t)}{\eta_{\mathit{c}}\,U_{\mathit{st}}(I_{\mathit{st}}(t))} \left(K\,R_{\mathit{em}}\,\mathit{u}(t) + \kappa_{\mathit{em}}\frac{\gamma}{\mathit{r_{w}}}\,\mathit{v}(t)\right)$$

-Implicit nonlinear static function

▶ Simpler model

$$\dot{m}_H(t) = b_0 + b_1 v(t) u(t) + b_2 u^2(t)$$

27/43

#### Outline

Repetition

Case study 6: Fuel Optimal Trajectories of a Racing FCEV Model compilation

#### Model simplification

Formulating the optimal control problem

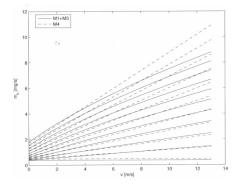
**Optimal Controllers Solutions** 

Final Results

Some Additional Material – Fuel Consumption

28/4

#### Simplified Fuel Consumption - Validation



29/43

#### Detour

- Occam's razor:
  - $-\mbox{The}$  explanation of any phenomenon should make as few assumptions as possible.
  - Shave of those who are unnecessary.
- Law of Parsimony: Among others a factor in statistics: In general, mathematical models with the smallest number of parameters are preferred as each parameter introduced into the model adds some uncertainty to it.
- Another viewpoint.
   Try to simplify the problem you s

Try to simplify the problem you solve as much as possible.

-Neglect effects and be proud when you are successful!

30/4

#### Outline

Repetition

Case study 6: Fuel Optimal Trajectories of a Racing FCEN Model compilation

Model simplification

#### Formulating the optimal control problem

**Optimal Controllers Solutions** 

Final Result

Some Additional Material – Fuel Consumptio

# **Optimal Control Problems**

Start of the cycle

$$v(0)=0, \qquad x(0)=0$$

$$\lambda_1(t_f) = 0, \qquad x(t_f) = x_f = v_m t_f$$

Periodic route

$$x(0) = 0$$

$$\lambda_1(t_f) = \lambda_1(0), \qquad x(t_f) = x_f = v_m t_f, \qquad v(t_f) = v(0)$$

43

Simple controller for the start

$$u(t) = K_p(f v_m - v(t)) + K_i \int_0^t (f v_m - v_t(t)) dt$$

f-tuning parameter  $\approx 1$  to allow for matching the average speed

#### Outline

Repetition

Case study 6: Fuel Optimal Trajectories of a Racing FCEV Model compilation

Model simplification

Formulating the optimal control problem

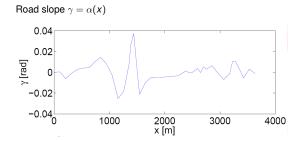
#### **Optimal Controllers Solutions**

Final Regulte

Some Additional Material – Fuel Consumption

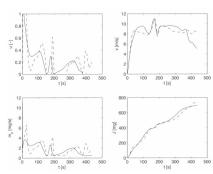
33/43 34/43

## Problem Setup - Road Slope Given



# Fuel Optimal Trajectory - Start

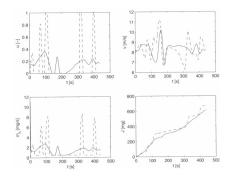
Fuel optimal trajectory has 7% lower fuel consumption



36/43

### Fuel Optimal Trajectory - Continuous Driving

Fuel optimal trajectory has 9% lower fuel consumption



Outline

Repetition

Case study 6: Fuel Optimal Trajectories of a Racing FCEV Model compilation

Model simplification

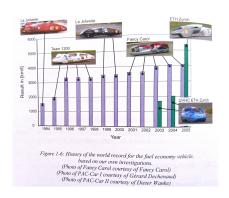
Formulating the optimal control problem

**Optimal Controllers Solutions** 

Final Results

Some Additional Material – Fuel Consumption

#### Final Results in Shell Eco Marathon



#### Outline

Repetition

Case study 6: Fuel Optimal Trajectories of a Racing FCEV Model compilation

Model simplification

Formulating the optimal control problem

**Optimal Controllers Solutions** 

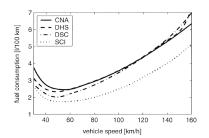
Final Results

Some Additional Material – Fuel Consumption

39/43 40/

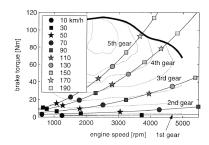
# Fuel Optimal Speed for Normal Driving

## ICE vehicle (light weight 800 kg)



# Engine Map and Gearbox Layout

# CI engine (light weight 800 kg)



41/43 42/43