

# Comparison of two Exhaust Manifold Pressure Estimation Methods

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## Abstract

*In turbocharged engines with wastegate the exhaust pressure can change rapidly. Two methods to estimate the exhaust manifold pressure are compared for diagnosis of wastegate and turbocharger of a spark-ignited engine. One relies on the first law of thermodynamics and produces changes in exhaust manifold pressure. The second uses a model of the mass of remaining exhaust gases in the cylinder and results in absolute estimations of the exhaust manifold pressure. They does not require any extra sensors in the exhaust system after the calibration. Estimates of the exhaust manifold pressure relies on information from an air-to-cylinder observer and a static map. The exhaust manifold pressure estimators are compared using a series of wastegate steps on a turbocharged SAAB 2.3 dm<sup>3</sup> SI-engine. The comparison showed that the method based on the first law of thermodynamics was best suited for diagnosis purposes since it was least sensitive to model errors.*

**Keywords:** Sensor fusion, exhaust pressure estimation

## 1. Introduction

Knowledge of the exhaust manifold pressure on a turbocharged SI-engine with wastegate is useful for diagnosis of the wastegate, the turbine, and the exhaust system. The wastegate controls the power to the turbine and prevents engine turbine destruction by reducing the pressure in the exhaust manifold. Therefore it is crucial for turbine safety to diagnose the wastegate. One method to diagnose wastegate operation is to use the exhaust manifold pressure. This is not normally measured due to the high temperatures in the exhaust system and the extra cost of an additional sensor. Estimators are therefore desirable for the exhaust manifold pressure. For naturally aspirated (NA) SI-engines estimators for pressure and temperature in the exhaust manifold have been proposed in [4] with good results. In NA engines the exhaust pressure is generated by the exhaust

system which acts as a constant flow restriction. On turbocharged engines with wastegate this restriction is made of two parts. One of them is the exhaust system which acts as a constant flow restriction and the second is the wastegate valve which shunts the exhaust gases past the restricting turbine. Changes in valve position therefore influences the flow restriction and the exhaust system can not be modeled as a constant flow restriction. Unfortunately the position of the wastegate is not measurable, which further complicates the situation.

The recently developed air-to-cylinder observer [1] offers a possibility to extract information about the cylinder conditions. One of the observer states is an estimate of the in-cylinder air mass offset compared to the expected air mass from the volumetric efficiency. In this paper the exhaust pressure is estimated with the use of the in-cylinder air mass offset and an additional static map.

Two different estimators are compared in their ability to estimate the exhaust manifold pressure and their sensitivity to model parameters. One is based on energy conservation [2] which estimates the change in exhaust pressure due to a change in air mass flow to the cylinder. In the second method the absolute exhaust manifold pressure is modeled when the change in air mass to cylinder is assumed to be the result of a change in remaining exhaust gas mass (residual gas).

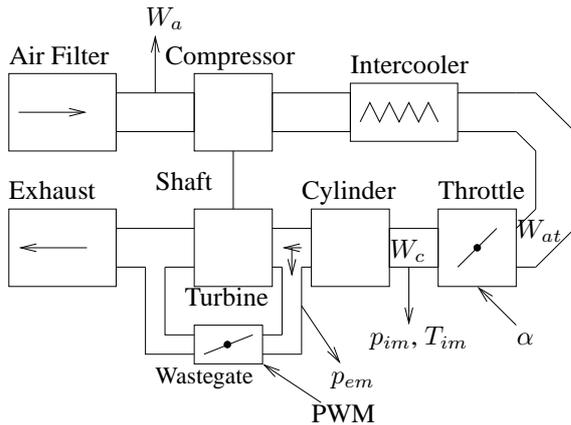
The estimated exhaust pressure can then for example be used for diagnosis of the wastegate, diagnosis of the turbine, or checking the back-pressure caused by the exhaust system. The developed estimator is nonlinear and model-based. No sensors in the exhaust system are needed by the observer after a calibration process. The sensors used are air mass flow, as well as temperature and pressure after the throttle, which are available on many production engines. The estimated exhaust pressure is only valid under steady-state conditions since the air-to-cylinder have to converge, a simplified intake manifold model with only pressure dynamics is used, and a wide-band air/fuel ratio sensor is used which have low pass characteristics.

## 1.1. System Overview

In Figure 1 the components of the engine and the sensors is shown. The air flows through the air-filter and is then measured by a hot-film air mass sensor  $W_a$ . It is then compressed and cooled by the intercooler, and the pressure  $p_{int}$ , and temperature  $T_{int}$  is measured. The air flow into the intake manifold is restricted by the throttle which is operated by setting the angle of the throttle plate  $\alpha$ . Air mass flow past the throttle and into the intake manifold is  $W_{at}$ . In the intake manifold there is one pressure sensor  $p_{im}$ , and one temperature sensor  $T_{im}$ .

From the intake manifold the air mass flow to the cylinders is  $W_c$  and it can only be measured stationary by  $W_a$ . The mass of air that can fill the cylinder depends on, among others, the amount of residual gases in the cylinder. The later is governed by the exhaust manifold pressure  $p_{em}$ , which in turn depends on the wastegate position. A closed wastegate increases the exhaust manifold pressure, and results in more residual gases and a smaller mass of air can fill the cylinder.

The wastegate is controlled by a pulse width modulated (PWM) signal.



**Figure 1. Sensors and actuators on the engine. The only measured air mass flow is before the compressor,  $W_a$ .**

## 2. Exhaust Pressure Models

Air mass to cylinder is influenced by the exhaust pressure and a model that describes the air mass to cylinder flow is therefore first described. Later two exhaust manifold pressure models are presented using information from the air to cylinder model. The intake manifold pressure dynamics is neglected when the exhaust manifold pressure is estimated.

Finally a brief summary of the exhaust manifold pressure estimators are given. For a description of the symbols used, please see the nomenclature at the end.

### 2.1. Air-to-cylinder Model

The standard method to model air to cylinder flow is to map the volumetric efficiency of the engine under stationary conditions [3, 5]. In the turbocharged engine the exhaust manifold pressure varies with the setting of the wastegate which affects the volumetric efficiency since it is a function of the pressure ratio  $\frac{p_{em}}{p_{im}}$ . This is supported by measurements, see Figure 2.

Changes in exhaust manifold pressure therefore influences the air mass that can enter the cylinder. Volumetric efficiency estimates the air mass to cylinder well if the exhaust manifold pressure is the same as during the engine mapping. However if the exhaust pressure is not the same as during the mapping there will be an offset, called  $m_{\Delta}$ , in estimated air compared to the actual air mass to the cylinder per combustion. Estimated air mass to cylinder is calculated using the volumetric efficiency of the engine,  $\eta_{vol}(N, p_{im}) \frac{p_{im} V_d}{R_c T_{im}}$ . Air mass flow is measured by a sensor and for each combustion the air mass is known at steady state  $W_a \frac{N}{n_r}$ .

$$m_{\Delta} = \eta_{vol}(N, p_{im}) \frac{p_{im} V_d}{R_c T_{im}} - W_a \frac{N}{n_r} \quad (1)$$

Stationary air mass offset  $m_{\Delta}$  is estimated by Equation (1) and can then be used to estimate the change in exhaust manifold pressure,  $p_{em_{\Delta}}$ , compared to the pressure during the mapping of volumetric efficiency. In [1] the air mass offset is also estimated under pressure transients.

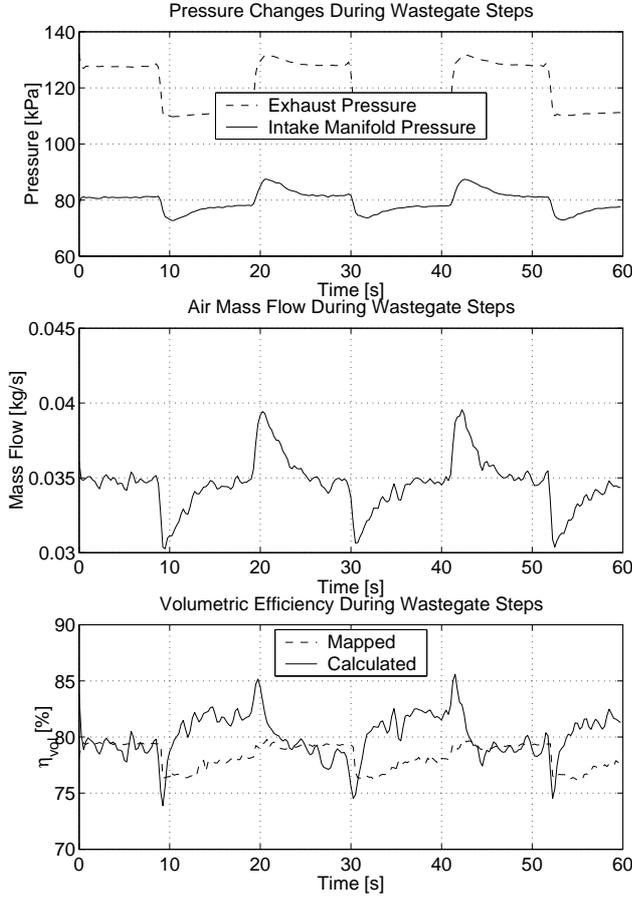
### 2.2. Exhaust Pressure Models

In Figure 3 the exhaust pressure are plotted under stationary conditions as a function of air mass offset, Equation (1).

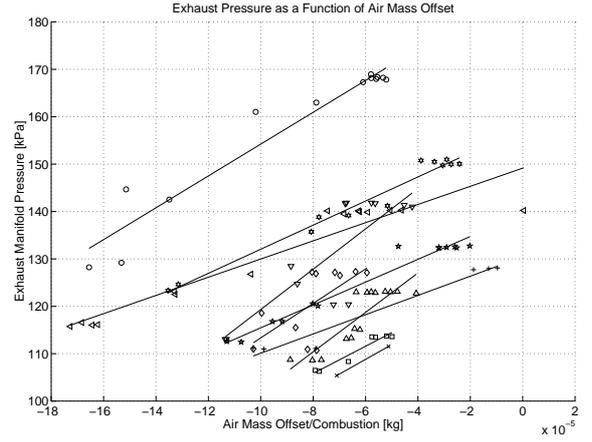
Two different models for exhaust manifold pressure estimation are introduced and compared. The first uses the first law of thermodynamics and estimates the change in exhaust manifold pressure due to a change in air mass to cylinder and is described in [2]. The second method estimates the absolute exhaust manifold pressure due to a change in residual gas mass which is approximated using the air mass to cylinder information.

#### 2.2.1. Energy Conservation

During the gas exchange, fresh gases are mixed with residual gases. If heat transfer to and from the walls are neglected, the internal energy of the mixture is conserved



**Figure 2. Top:** When the wastegate is opened the exhaust pressure drops but the air mass flow is maintained constant by a controller (**center**). **Bottom:** At stationary conditions with closed wastegate (time 8, 28, and 50) the mapped volumetric efficiency agrees with the current volumetric efficiency  $\eta_{vol} = \frac{W_{af} RT_{im} n_r}{p_{im} V_d N}$ . When the wastegate is open (time 17, 30, and 60) the volumetric efficiency does not match the mapped value.



**Figure 3. Measured exhaust pressure with different wastegate settings when the engine was ran stationary. The exhaust pressure is linear as a function of the air mass offset  $m$ . The slope of the fitted first order function varies with engine operating point.**

according to the first law of thermodynamics. A standard assumption [3] is to assume constant specific heat  $c_v$  for unburned and burned mixture.

$$m c_v T_1 = m_{af} c_v T_{af} + m_r c_v T_r \quad (2)$$

If the gases are assumed to be ideal the total in cylinder mass  $m$  and the residual gas mass  $m_r$  can be obtained.

$$m = \frac{p_{im} (V_c + V_d)}{RT_{im}} \quad (3)$$

$$m_r = \frac{p_{em} V_r}{RT_r} \quad (4)$$

The exhaust manifold pressure can be calculated by inserting Equations (3, 4) into Equation (2), given the volume of the residual gases  $V_r$  and their temperature  $T_r$ .

$$m_{af} = m_a + m_f = m_a \left( 1 + \frac{1}{\lambda \left( \frac{A}{F} \right)_s} \right)_k$$

$$p_{em} = \frac{V_c + V_d}{V_r} p_{im} - \frac{R}{V_r} m_{af} T_{im} = \underbrace{\frac{V_c + V_d}{V_r} \cdot p_{im}}_{p_{exhmap}(N, p_{im})} - \underbrace{\eta_{vol} p_{im} V_d k}_{p_{em\Delta}(m_{\Delta}, R(\lambda), T_{im}, k)} - \frac{R}{V_r} m_{\Delta} T_{im} k \quad (5)$$

In Equation (5)  $\frac{V_c+V_d}{V_r}$  is constant since the engine does not have variable valve timing. In  $p_{\text{exh}_{\text{map}}}(N, p_{\text{im}})$  the second term

$$\eta_{\text{vol}} p_{\text{im}} V_d \underbrace{\left(1 + \frac{1}{\lambda \left(\frac{A}{F}\right)_s}\right)}_k$$

can be regarded as independent of  $\lambda$ . Volumetric efficiency is quasi-statically a product where one factor [3] is

$$\frac{1}{1 + \frac{1}{\lambda \left(\frac{A}{F}\right)_s}} = \frac{1}{k}$$

which cancels the dependency of the air/fuel ratio. The static part of the exhaust manifold pressure,  $p_{\text{exh}_{\text{map}}}(N, p_{\text{im}})$ , is determined during engine mapping. Gas constant  $R$  is also a function of air/fuel ratio and is not equal for burned and unburned mixture. The amount of residual gases  $m_r$  is small compared to the mass of air and fuel  $m_{af}$  and therefore the gas constant for the unburned mixture is used and the impact of residual gases are neglected. Air/fuel ratio influences the gas constant in the following way

$$R(\lambda) = \frac{\tilde{R}}{\mathcal{M}} = \frac{\tilde{R}}{\frac{m_{\text{fuel}} + m_{\text{air}}}{n_{\text{fuel}} + n_{\text{air}}}} = \frac{\tilde{R} \left(1 + \lambda \left(\frac{A}{F}\right)_s\right)}{\mathcal{M}_{\text{fuel}} + \lambda \mathcal{M}_{\text{air}} \left(\frac{A}{F}\right)_s} \quad (6)$$

Isooctane was used as fuel data in Equation (6) and  $R(\lambda)$  was inserted into Equation (5). If there is no sensor data available of the air/fuel ratio it can be approximated using the relationship

$$\lambda = \frac{W_{at}}{\left(\frac{A}{F}\right)_s \dot{m}_f}$$

Fuel mass flow is calculated using the injector approximation:

$$\dot{m}_f = \frac{N}{n_r} n_{\text{cyl}} K_{inj} (t_{inj} - t_0)$$

The injector pulse width  $t_{inj}$  is available in most ECUs.

The resulting exhaust manifold pressure model has only one parameter and that is the volume of the residual gases  $V_r$  it was estimated using a least-square method on measured engine data.

### 2.2.2. Residual Gas Mass Estimation

The exhaust pressure is modeled based on estimated residual gas mass. To calculate the residual gas mass, an ideal otto-cycle is assumed with no heat transfer and no crevice volumes. The residual gas mass fraction is defined  $x_r = \frac{m_r}{m_t}$ , where  $m_r$  is the residual gas mass and the total in cylinder mass is  $m_t = m_r + m_{af}$ . The sum of air mass and

the mass of fuel is  $m_{af} = W_c \left(1 + \frac{1}{\lambda}\right) \frac{2}{N}$ . The air/fuel ratio  $\frac{A}{F} = \lambda \left(\frac{A}{F}\right)_s$ . The specific internal energy of the gas in the cylinder with the mass  $m_t$  is

$$Q_{in} = \frac{m_f Q_{HV}}{m_t} \quad (7)$$

The temperature of the gases inside the cylinder at intake valve closing (IVC) is  $T_1 = (1 - x_r) T_{im} + T_r$ . With the assumptions of ideal otto cycle and ideal gases yields the following model [3] for the exhaust pressure

$$p_{em} = p_{im} (x_r r_c)^\gamma \left(1 + \frac{Q_{in}}{c_v T_1 r_c^{\gamma-1}}\right) \quad (8)$$

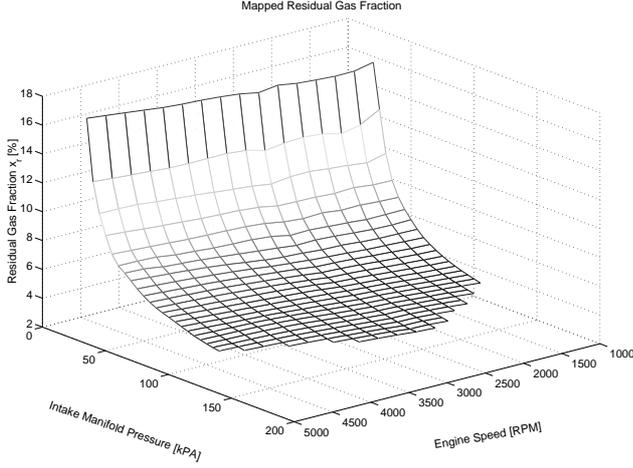
In Equation (8), the residual gas mass fraction  $x_r$  is needed. To calculate it the nominal residual gas mass is stored as a map of the engine speed and the intake manifold pressure. The air-to-cylinder observer estimates the offset in air mass flow to the cylinder in the state  $m_\Delta$ , Equation (1). Using a simplified model of the residual gas fraction  $x_r$  it can be calculated as

$$x_r = \frac{m + m_{r_{\text{map}}}}{m_{af} + m + m_{r_{\text{map}}}} \quad (9)$$

The residual gas mass in the cylinder is the sum of the mapped residual gas mass  $m_{r_{\text{map}}}$  and the in cylinder air mass offset  $m_\Delta$ . The value of  $m_{r_{\text{map}}}$  was calculated from steady state measurements using fix point iteration to find the residual gas fraction  $x_r$

1. Initiate  $x_r = 0$
2. If  $x_r > 0$  then  $T_1 = (1 - x_r) T_{im} + x_r T_r$  else  $T_1 = T_{im}$ . The temperature in the intake manifold  $T_{im}$  is measured.
3. If  $x_r > 0$  then  $T_r = T_1 \left(\frac{p_{em}}{p_{im}}\right)^{1-\frac{1}{\gamma}} \left(1 + \frac{Q_{in}}{c_v T_1 r_c^{\gamma-1}}\right)^{\frac{1}{\gamma}}$ . Exhaust pressure  $p_{em}$  is measured during engine mapping. The specific heat  $Q_{in}$  have to be updated each iteration according to Equation (7).
4. Calculate  $x_r = \frac{1}{r_c} \left(\frac{p_{em}}{p_{im}}\right)^{\frac{1}{\gamma}} \left(1 + \frac{Q_{in}}{c_v T_1 r_c^{\gamma-1}}\right)^{-\frac{1}{\gamma}}$ . If the new value of  $x_r$  differs more than  $\epsilon$  then the iteration continues with the this  $x_r$  from step 2.

The algorithm is based on equation in [3]. The calculated residual gas fraction is then converted to residual gas mass  $m_{r_{\text{map}}} = \frac{x_r}{1-x_r} m_{af}$ . The resulting  $m_{r_{\text{map}}}$  is then stored as a two-dimensional map of engine speed and intake manifold pressure. See Figure 4 for the calculated  $x_r$  from the engine mapping. It is calculated in the same points as the volumetric efficiency and stored as a function of engine speed and intake manifold pressure.



**Figure 4. Mapped residual gas fraction. The mass  $m_r$  is stored in the map used, but the residual gas fraction is shown since it is a more intuitive measure of residual gas.**

In Equation (8) the temperature  $T_r$  and the exhaust pressure  $p_{em}$  is calculated using the same iteration principle as for the mapping of  $m_r$  except for that the  $p_{em}$  is solved for instead of the residual gas mass fraction. Unfortunately the algorithm does not always converge when estimating the exhaust pressure. Divergence occurs e.g. when the estimated residual gas mass is negative, which is impossible in the real engine cycle.

### 2.3. Summary of Estimators

The estimation process of each method is briefly summarized here.

#### 2.3.1. Energy Conservation

First the air mass offset  $m_\Delta$ , Equation (10b), is calculated. The air fuel ratio dependency is captured by Equation (10c) and is inserted into the final equation 10d.

$$R_c(\lambda) = \frac{\tilde{R}_c \left(1 + \lambda \left(\frac{A}{F}\right)_s\right)}{\mathcal{M}_{fuel} + \lambda \mathcal{M}_{air} \left(\frac{A}{F}\right)_s} \quad (10a)$$

$$m_\Delta = \eta_{vol}(N, p_{im}) \frac{p_{im} V_d}{R_c(\lambda) T_{im}} - W_a \frac{n_r}{N} \quad (10b)$$

$$k = 1 + \frac{1}{\lambda \left(\frac{A}{F}\right)_s} \quad (10c)$$

$$p_{em} = p_{em_{map}}(N, p_{im}) - \frac{R_c(\lambda)}{V_r} m_\Delta T_{im} k \quad (10d)$$

#### 2.3.2. Residual gas Mass Estimation

In this method iteration is used to estimate the absolute pressure in the exhaust manifold,  $p_{em}$ . Three variables are sought,  $p_{em}$ , temperature at start of compression  $T_1$ , and residual gas temperature  $T_r$ . Iterate Equations (11e, 11f, and 11g). Initial values are necessary for two of the equations.  $p_{em}$  from a map  $p_{exh_{map}}(N, p_{im})$  and  $T_r = 1400$ .

$$m_\Delta = \eta_{vol}(N, p_{im}) \frac{p_{im} V_d}{R T_{im}} - W_a \frac{n_r}{N} \quad (11a)$$

$$m_t = m_{af} + m_\Delta + m_{r_{map}} \quad (11b)$$

$$x_r = \frac{m_\Delta + m_{r_{map}}}{m_t} \quad (11c)$$

$$Q_{in} = \frac{m_f Q_{HV}}{m_t} \quad (11d)$$

$$T_1 = (1 - x_r) T_{im} + x_r T_r \quad (11e)$$

$$T_r = T_1 \left(\frac{p_{em}}{p_{im}}\right)^{1-\frac{1}{\gamma}} \left(1 + \frac{Q_{in}}{c_v T_1 r_c^{\gamma-1}}\right)^{\frac{1}{\gamma}} \quad (11f)$$

$$p_{em} = p_{im} (x_r r_c)^\gamma \left(1 + \frac{Q_{in}}{c_v T_1 r_c^{\gamma-1}}\right) \quad (11g)$$

### 3. Measurement Setup

The measurements were performed on a 2.3 dm<sup>3</sup> turbocharged SAAB spark ignition engine with wastegate and drive-by-wire system. The engine is connected to an asynchronous Dynas 220 NT dynamometer, which is operated at constant speed mode. The dynamometer is controlled by a PC and the engine is controlled by a research engine management system called Trionic 7. The engine management unit was connected to a PC in the control room using a CAN-bus. From the control room it is possible to control the throttle and the wastegate. The later was also manually operated with a handle.

The engine is equipped with additional pressure sensors Kristall 4293A2 and Kristall 4293A5 before the throttle, in the intake manifold, and in the exhaust manifold before the turbine. There are also extra temperature sensors of PT200 type, Heraeus ECO-TS200s, in the intake manifold and between the intercooler and the throttle and in the exhaust manifold close to the turbine.

All measurements were performed with an HPE 1415A, which is a VXI-instrument. Engine mapping was performed with a sampling frequency of 10 Hz and the signals were low-pass filtered at 5 Hz to avoid aliasing. The engine mapping was performed from 1000 RPM up to 4800 RPM in steps of approximately 500 RPM. The lower limit was due to severe vibrations at higher loads. The engine was run 25 seconds in each work point before a 5 second sampling was started. The median of the sampled data was then stored in 114 points.

Step response experiments were performed with the same instrument HPE 1415A and a sampling frequency of 1 kHz was used. Anti-alias filters were disabled due to the damping and delay introduced by the filter.

## 4. Comparison of Estimators

In the comparison process measured engine data was used. The test cases will be described thoroughly in the next section. The exhaust pressure is estimated using the static map of the exhaust manifold pressure and air mass offset information  $m_{\Delta}$ .

### 4.1. Description of Test Cases

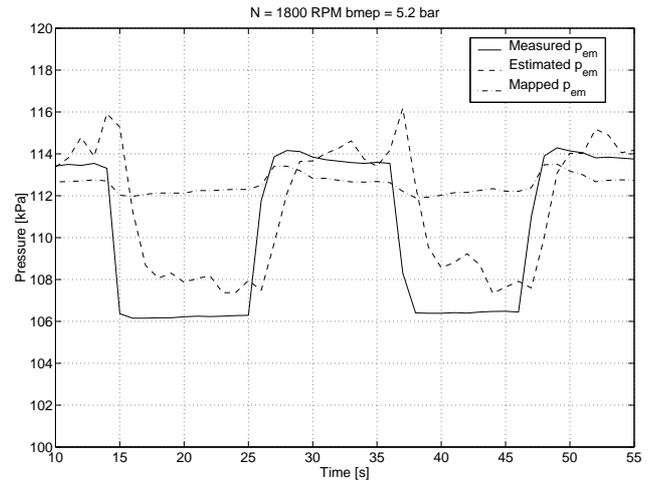
The estimators are validated using measurements of the exhaust pressure while wastegate valve was manually operated. In the engine management system, a controller tried to maintain constant air mass flow through the throttle. Since the power to the compressor is reduced when the wastegate is opened the throttle controller will open the throttle to compensate for the lowered air mass flow. Throttle angle will therefore not be constant during the test, which affects the air mass flow through the throttle and the air dynamics introduces a small deviation in the estimated air mass offset  $m_{\Delta}$  until the system has settled.

### 4.2. Estimated Exhaust Pressure

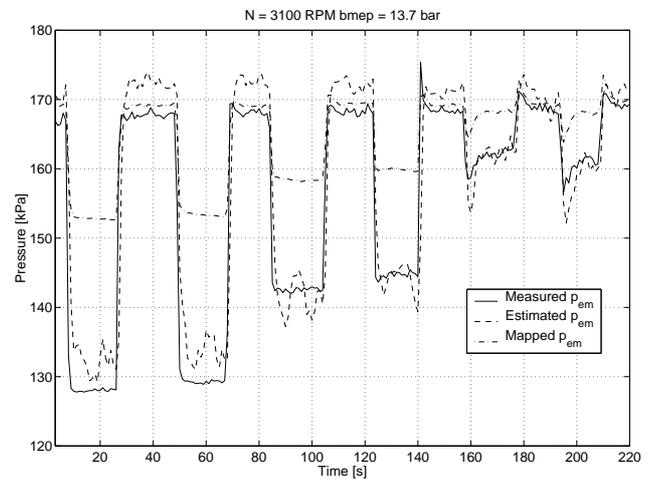
Measurements have been taken for a number of engine speeds between 1600 and 3100 RPM. In each measurement the engine speed was held constant and the wastegate was initially controlled by the ECU. The wastegate was opened and held constant for approximately 10 seconds and then closed.

#### 4.2.1. Energy Conservation

To show the dynamic behavior of the exhaust manifold pressure observer two operating points was chosen, one at low engine speed and load and one at higher engine speed and higher load. To reduce noise the signals used in the computation of Equation (5) have been low pass filtered and so have the measured exhaust manifold pressure been to reduce engine pumping fluctuations. The low speed and load case is shown in Figure 5 and the higher speed and load is shown in Figure 6 where different settings of the wastegate was used. It takes a few seconds for the estimated exhaust manifold pressure to converge since stationary conditions have been assumed to calculate the air mass offset  $m_{\Delta}$ . The low pass filtering of e.g. the air mass flow signal also delays the estimated pressure.



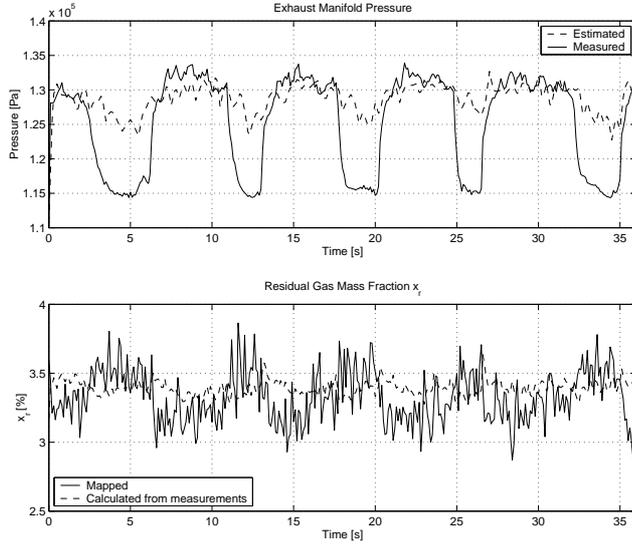
**Figure 5. Estimated exhaust manifold pressure during a wastegate step at low engine speed and load. Steady-state performance is within a few percent.**



**Figure 6. Estimated exhaust manifold pressure during a wastegate step at medium engine speed and higher load.**

#### 4.2.2. Residual Gas Mass Estimation

If only the static map of the residual gas mass is used the estimate of the exhaust pressure, Equation (8), is not able to capture the drop in exhaust pressure when the waste-gate valve is opened. This is due to that the static map which over estimates the residual gas amount. See Figure 7.



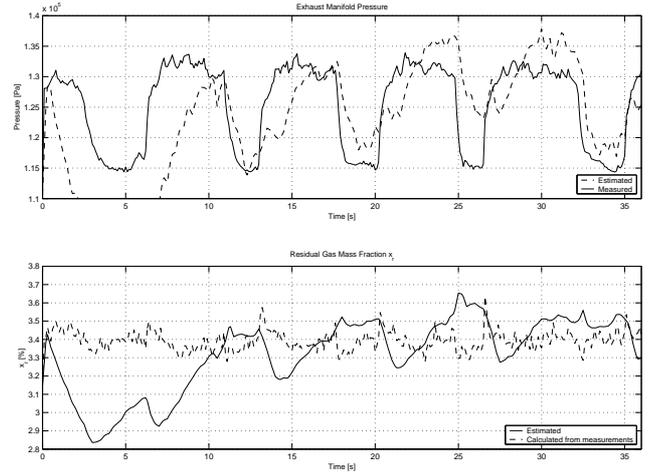
**Figure 7. Top: Exhaust pressure estimation without the use of the estimated residual gas mass. The estimate is within a few percent when the waste-gate is closed, that is when the exhaust pressure is approximately 130 kPa.**

**Bottom: Residual gas mass fractions from mapped values and calculated from test case. Note that the mapped  $x_r$  is higher when the waste gate is open, which explains why the estimated exhaust pressure is higher than the measured.**

When the information of the mass offset from the air-to-cylinder observer is used together with the static map of the residual gas mass in Equation (8) the estimated exhaust pressure can capture the openings of the waste-gate. See Figure 8. Note how the estimated residual gas fraction varies as the waste-gate is operated.

#### 4.3. Parameter Sensitivity

The two estimators have a different number of parameters. An interesting question is how sensitive the estimate is to errors in parameters.



**Figure 8. Top: Estimated and measured exhaust pressure. It takes 10 seconds for the estimator to converge.**

**Bottom: Estimated residual gas mass fraction with information from the air-to-cylinder observer. Note how the estimated residual gas mass fraction changes when the waste-gate is operated. It rises when the waste-gate is closed and falls when it is opened.**

#### 4.3.1. Energy Conservation

If the absolute exhaust pressure is estimated there will obviously be an offset if the mapped pressure is incorrect. Since the estimated exhaust manifold pressure is a sum of a mapped value and an estimated offset  $p_{em\Delta}$ . The later depends on  $m_\Delta$  which is calculated in Equation (1), measured intake manifold temperature, air/fuel ratio, and estimated volume of the residual gases  $V_r$ . The  $m_\Delta$  is affected by errors in the measured air mass flow into the manifold  $W_{at}$ ,  $p_{im}$ , and volumetric efficiency  $\eta_{vol}$ .

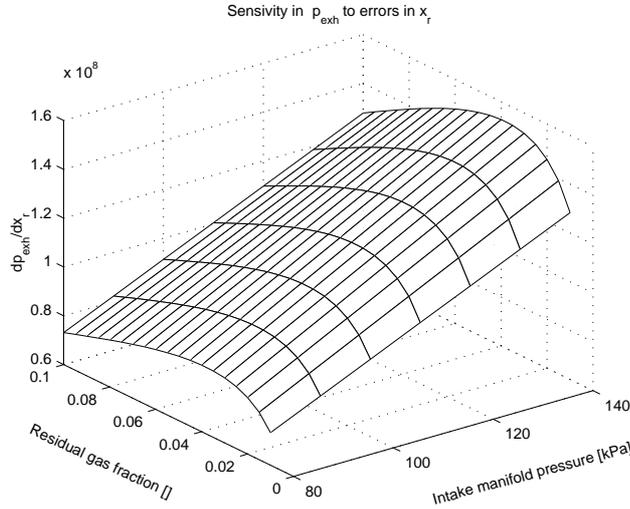
Residual gas volume  $V_r$  is assumed to be constant but in reality it is not since there are dynamic effects in the gas exchange such as the inertia of the gases.

#### 4.3.2. Residual Gas Mass Estimation

There are two maps in the model, one for volumetric efficiency  $\eta_{vol}$  and one for the nominal residual gas mass  $m_r$ . If there is an error in the volumetric efficiency map it will result in an error in the estimated mass offset  $\Delta m$ . Suppose that there is an offset in  $\eta_{vol} = \eta_{vol_0} + \Delta\eta_{vol}$ . This will result in an estimated mass offset  $m_\Delta = \Delta\eta_{vol} \frac{p_{im} V_d}{R_{im} T_{im}}$  and it will affect the residual gas estimate Equation (9). Errors in the air flow  $W_{at}$  into the intake manifold will have a similar

effect.

As discussed previously errors in throttle model and volumetric efficiency map both affect the estimated residual gas mass and therefore  $x_r$ . A study of  $\frac{dp_{em}}{dx_r}$  is shown in Figure 9, where the impact of an error in  $x_r$  is greater for larger intake manifold pressures.



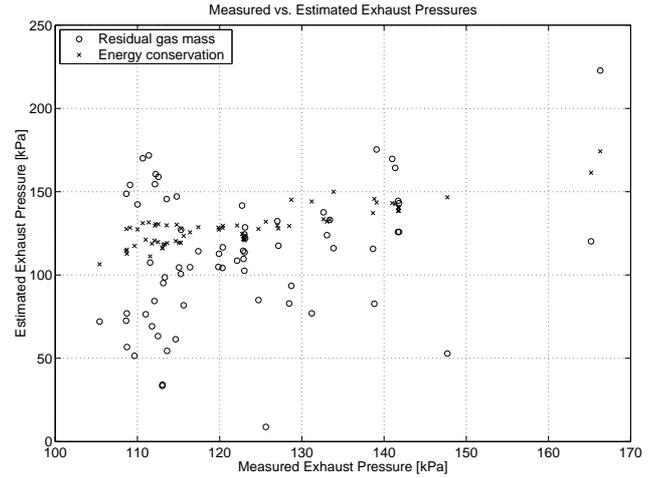
**Figure 9.**  $\frac{dp_{em}}{dx_r}$  for different points  $(p_{im}, x_r)$ . In the calculations  $c_v$ ,  $\eta_{vol}$ ,  $T_{im}$ ,  $\gamma$ , and  $T_r$  were fixed. The sensitivity to errors in  $x_r$  are linear in intake manifold pressure and almost constant for varying  $x_r$ .

#### 4.4. Summary of Comparison

The estimator based on energy conservation is more robust than the residual gas mass based estimator since it estimates changes in exhaust manifold pressure. The estimator based on residual gas mass estimation is more sensitive to biases in air mass offset  $m_{\Delta}$ . Even for small errors it might not converge. Another drawback is that the sensitivity for errors in estimated residual gas fraction  $x_r$  increases for larger intake manifold pressures. This is unfortunate since the wastegate is usually operated for larger intake manifold pressures, as shown in Figure 9.

At steady-state the methods have been compared using different openings of the wastegate. Energy conservation method had the least root mean square error as shown in Figure 10.

Another advantage of the energy conservation based method is its low computational complexity compared to the residual gas mass based method. In the later the number of iteration required is not fix which makes it hard to predict when the estimate is ready.



**Figure 10.** Stationary evaluation of the estimated exhaust pressure compared to measured pressure. Residual gas mass estimation results in a higher root square mean square error 4.7 kPa compared to 1.1 kPa for the energy conservation method.

## 5. Conclusions

Two methods to estimate the exhaust manifold pressure have been compared. The estimator based on energy conservation, the first law of thermodynamics, had few parameters and estimated changes in exhaust manifold pressure due to changes in air mass offset  $m_{\Delta}$  well. The second estimate using estimates of residual gas mass is more sensitive to errors in the air mass offset estimation since it produces an absolute exhaust pressure.

## Acknowledgments

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## Nomenclature

Symbol	Description
$p_{em}$	Exhaust manifold pressure
$p_{em\Delta}$	Exhaust manifold pressure change from expected (mapped) pressure
$p_{im}$	Intake manifold pressure
$T_{im}$	Intake manifold temperature
$T_r$	Temperature of residual gases
$T_1$	Temperature of air/fuel charge at start of compression
$\eta_{vol}$	Volumetric efficiency
$R_{im}$	Specific gas constant in the intake manifold
$\alpha$	Throttle angle
$W_{at}$	Air mass flow through throttle
$W_a$	Measured air mass flow
$W_c$	Air mass flow to cylinder
$m_{af}$	Mass of air and fuel in the cylinder
$m_{\Delta}$	Air mass to cylinder offset, calculated using mapped volumetric efficiency.
$\mathcal{M}_{air}$	Molecular weight of air
$\mathcal{M}_{fuel}$	Molecular weight of fuel
$n_{air}$	Number of moles of air
$n_{fuel}$	Number of moles of fuel
$c_v$	Specific heat at constant volume
$\gamma$	Ratio of specific heats
$x_r$	Residual gas fraction
$m_r$	Residual gas mass
$r_c$	Compression ratio of the engine
$\lambda$	Normalized air/fuel ratio
$(\frac{A}{F})_s$	Stoichiometric air/fuel ratio
$k$	Scaling factor to calculate air and fuel mass given air mass, $k = 1 + \frac{1}{\lambda(\frac{A}{F})_s}$
$V_d$	Displacement volume
$V_c$	Clearance volume
$V_r$	Volume of residual gases
$V_{im}$	Volume of intake manifold
$n_r$	Number of revolutions per cycle
$N$	Engine speed i revolutions per second
$\tilde{R}$	Gas constant, 8.31