A DERIVATION OF THE MINIMAL POLYNOMIAL BASIS APPROACH TO LINEAR RESIDUAL GENERATION

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Abstract:
A fundamental part of a fault diagnosis system is the residual generator. Here a new method, the *minimal polynomial basis approach*, for design of residual generators for linear systems, is presented. The residual generation problem is transformed into a problem of finding polynomial bases for null-spaces of polynomial matrices. This is a standard problem in established linear systems theory, which means that numerically efficient computational tools are generally available. It is shown that the minimal polynomial basis approach can find all possible residual generators and explicitly those of minimal order.

Keywords: fault detection, diagnosis, polynomial methods, decoupling, disturbance rejection

1. INTRODUCTION

The task of fault diagnosis is to, from known signals, i.e. measurements and control signals, detect and locate any faults acting on the system being supervised. A fundamental part of a *model based* diagnosis system is the *residual generator*. The residual generator filters known signals and generates a signal, the *residual*, that should be small (ideally 0) in the fault-free case and large when a fault is acting on the system.

This work is a study of *linear* residual generation for *linear* systems with no model uncertainties where any faults and disturbances acting on the system are modeled as input *signals*. To be able to produce a correct diagnosis in all operating conditions, influence from all disturbances on the residual need to be decoupled. Also, to facilitate fault isolation, not only disturbances need to be decoupled, but also a subset of the faults. By generating a set of such scalar residuals where different subsets of faults are decoupled in each residual, fault isolation is possible. Therefore it is convenient to distinguish between *monitored* and *non-monitored* faults. Monitored faults are the fault signals that the residual should be sensitive to. Non-monitored faults are the fault signals that the residual should not be sensitive to, i.e. the faults that are to be decoupled. With this approach, the design of a residual generator becomes a decoupling problem.

A number of design methods for designing linear residual generators have been proposed in literature, see for example (Chen & Patton 1999, Wünnenberg 1990, Massoumnia, Verghese & Willsky 1989, Nikoukhah 1994, Chow & Willsky 1984, Nyberg & Nielsen 2000). Natural questions that have not gained very much attention before are the following:

- Does the method find all possible residual generators?
- Does the method find residual generators of minimal order

Now follows a brief motivation to why low and minimal order residual generators are interesting.

1.1 *Why low-order Residual Generators?*

Stochastic descriptions of measurement noise and model uncertainties is not considered in this paper. For such stochastic systems, low-order residual generators may have suboptimal properties. Compare reduced observers with full order observers. Solutions for the stochastic problem is covered in e.g. (Nikoukhah 1994).

The reason for the interest in the low and minimal order properties of the residual generator is primarily that we want to depend on the model as little as possible. A low order usually implies that only a small part of the model is utilized, i.e. *local* relationships in the model is utilized. Since all parts of the model have errors, this further means that few model errors will affect the residual. Also, lower complexity of the residual generator means easier implementation and less on-line computational burden.

The following small example will highlight this issue. Consider a linear system with two sensors, one actuator, and a modeled sensor fault in the second sensor.

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{s+a} \\ \dfrac{1}{(s+b)(s+a)} \end{pmatrix} u + \begin{pmatrix} 0 \\ 1 \end{pmatrix} f$$

The model consists of two model parameters, a and b. To detect the fault, a second order residual

$$r_1 = y_2 - \frac{1}{(s+a)(s+b)} u$$

can be used. Examining the expression gives that the residual relies on the accuracy of both model parameters a and b. Using straightforward manipulations of model equations, it is possible to derive a new, *first order* residual representing a local relationship between the two sensor signals:

$$r_2 = \frac{1}{s+b} y_1 - y_2$$

As can be seen, residual r_2 only depends on the accuracy of parameter b. Thus, a lower order residual generator resulted in a residual generator less dependent on the model accuracy. Here, in this example, even complete invariance of model accuracy of parameter a was achieved. This is not a general result, model dependency does not always decrease with the order. However, if the model has such a property, systematic utilization of low-order residual generators is desirable.

2. POLYNOMIAL BASES AND RATIONAL VECTOR SPACES

This paper relies on established theory on polynomial matrices, rational vector spaces, and polynomial bases for these spaces (Kailath 1980, Forney 1975, Chen 1984). The main notions used are presented in this section.

The *row-degree* of a row vector of polynomials is defined as the largest polynomial degree in the row-vector. In this paper, *polynomial bases* and *orders* of polynomial bases are of special interest. A polynomial basis is here represented by a polynomial matrix where the rows are the basis vectors. The *order* of a polynomial basis $F(s)$ is defined as the sum of the its row-degrees. A *minimal polynomial basis* for a rational vector-space $\mathcal{F}$ is then any polynomial basis that minimizes this order.

A matrix $F(s)$ is *irreducible* if and only if $F(s)$ has full rank for all s. A matrix $F(s)$ can always be written as

$$F(s) = S(s)D_{hr} + L(s)$$

where $S(s) = \text{diag}\{s^{\mu_i}, i = 1, \ldots, p\}$ where μ_i is the row-degrees of $F(s)$ and D_{hr} is the *highest-row-degree coefficient matrix*. A matrix is *row-reduced* if its highest-row-degree coefficient matrix D_{hr} has full row rank.

In addition to these definitions, the following theorem will be used:

Theorem 1. (Kailath,1980, Theorem 6.5-10). The rows of a matrix $F(s)$ form a minimal polynomial basis for the rational vector space they generate, if and only if $F(s)$ is irreducible and row-reduced.

3. THE MINIMAL POLYNOMIAL BASIS APPROACH

This section introduces the *minimal polynomial basis approach* to the design of linear residual generators. All derivations are performed in the continuous case but the corresponding results for the time-discrete case can be obtained by substituting s by z and *improper* by *non-causal*.

3.1 *Problem Formulation*

This section introduces the problem formulation which has been addressed in many papers, e.g. (Gertler 1991).

The systems studied here are assumed to be on the form

$$y = G(s)u + H(s)d + L(s)f \tag{1}$$

where y is measurements, u is known inputs to the system, d is disturbances including non-monitored faults, and f is the monitored faults. A general linear residual generator can be written

$$r = Q(s) \begin{pmatrix} y \\ u \end{pmatrix} \tag{2}$$

i.e. $Q(s)$ is a multi-dimensional transfer matrix with known signals y and u as inputs and a *residual* as output. A *stable* filter $Q(s)$ in (2) is a residual generator if and only if $r = 0$ for all d and u when $f = 0$. To be able to detect faults, it is also required that $r \neq 0$ when $f \neq 0$.

3.2 *Derivation of Design Methodology*

Inserting (1) into (2) gives

$$r = Q(s)\begin{bmatrix} G(s) & H(s) \\ I & 0 \end{bmatrix}\begin{bmatrix} u \\ d \end{bmatrix} + Q(s)\begin{bmatrix} L(s) \\ 0 \end{bmatrix} f$$

To make $r(t) = 0$ when $f(t) = 0$, it is required that disturbances and the control signal are *decoupled*, i.e. for $Q(s)$ to be a residual generator, it must hold that

$$Q(s)\begin{bmatrix} G(s) & H(s) \\ I & 0 \end{bmatrix} = 0$$

This implies that $Q(s)$ must belong to the left null-space of

$$M(s) = \begin{bmatrix} G(s) & H(s) \\ I & 0 \end{bmatrix} \tag{3}$$

This null-space is denoted $\mathcal{N}_L(M(s))$. The matrix $Q(s)$ need to fulfill two requirements: belong to the left null-space of $M(s)$ *and* give good fault sensitivity properties. If, in the first step of the design, *all* $Q(s)$ that fulfills the first requirement is found, then in a second step a single $Q(s)$ with good fault sensitivity properties can be selected. Thus, in a first step of the design, f or $L(s)$ need not be considered. The problem is then to find *all* rational $Q(s) \in \mathcal{N}_L(M(s))$. Of special interest are residual generators of low order for reasons discussed in Section 1.

Thus, a procedure to find all $Q(s) \in \mathcal{N}_L(M(s))$, and explicitly those of minimal order, is wanted. This can be done by finding a *minimal polynomial basis* for the rational vector-space $\mathcal{N}_L(M(s))$. Procedures for doing this will be described in Section 4. For now, assume that such a basis has been found and is formed by the rows of a matrix denoted $N_M(s)$. By inspection of (3), it can be realized that the dimension of $\mathcal{N}_L(M(s))$ (i.e. the number of rows of $N_M(s)$) is

$$\text{Dim } \mathcal{N}_L(M(s)) = m - \text{Rank}\, H(s) \overset{*}{=} m - k_d \tag{4}$$

where m is the dimension of y, k_u is the dimension of u, and k_d is the dimension of d. The last equality, marked $\overset{*}{=}$, holds only if Rank $H(s) = k_d$, but this should be the normal case.

Forming the Residual Generator

The second and final design-step is to use the polynomial basis $N_M(s)$ to form the residual generator. For this, consider the following theorem:

Theorem 2. (Kailath,1980, p.401). If the rows of $F(s)$ form an irreducible polynomial basis for a rational vector space $\mathcal{F}$, then all polynomial row vectors $x(s) \in \mathcal{F}$ can be written $x(s) = \phi(s)F(s)$ where $\phi(s)$ is a *polynomial* row vector.

The minimal polynomial basis $N_M(s)$ is irreducible according to Theorem 1, and then according to Theorem 2, all decoupling *polynomial* vectors $F(s)$ can be parameterized as

$$F(s) = \phi(s)N_M(s) \tag{5}$$

where $\phi(s)$ is a polynomial vector of suitable dimension. The row-vector $F(s)$ corresponds to a parity equation $F(s)\left(\begin{smallmatrix} y \\ u \end{smallmatrix}\right)$=0, (Gertler 1991). Since $N_M(s)$ is a basis, the parameterization vector $\phi(s)$ have minimal number of elements, i.e. $N_M(s)$ gives a minimal parameterization of *all* parity functions, not only minimal order. This is not accomplished with e.g. iterative parity space methods as described in Section 6.1.

One of the rows of $N_M(s)$ corresponds to a decoupling polynomial vector of minimal row-degree. The reason for this can be explained as follows. Consider a basis $N_M(s)$ with three rows and the row-degrees are d_1, d_2, and d_3 respectively. Since $N_M(s)$ is a minimal polynomial basis, $d_1 + d_2 + d_3$ is minimal. Now assume that the minimal row-degree of any decoupling vector is d_{min} and that $d_{min} < d_i$ for all d_i. Then by using a minimal row-degree decoupling vector, a new basis with less row-degree can be obtained. Thus $N_M(s)$ can not be a minimal basis, which shows that one of the rows of $N_M(s)$ must correspond to a decoupling vector of minimal row-degree.

A realizable rational transfer function $Q(s)$, i.e. the residual generator, can be found as

$$Q(s) = c^{-1}(s)F(s) \tag{6}$$

where the scalar polynomial $c(s)$ has greater or equal degree compared to the row-degree of $F(s)$. The degree constraint is the only constraint on $c(s)$. This means that the dynamics, i.e. poles, of the residual generator $Q(s)$ can be chosen freely, e.g. to impose a low-pass characteristic of the residual generator to filter out noise or high frequency uncertainties. Thus, $\phi(s)$ and $c(s)$ includes all design freedom that can for example be used to shape the fault-to-residual response. This also means that the order of a realization of a residual generator is determined by the row-degree of the polynomial vector $F(s)$.

4. METHODS TO FIND A MINIMAL POLYNOMIAL BASIS TO $\mathcal{N}_\mathbf{L}(\mathbf{M}(\mathbf{s}))$

The problem of finding a minimal polynomial basis to the left null-space of the matrix rational $M(s)$ can be solved by transforming this to the problem of finding a polynomial basis for the null-space of a *polynomial* matrix. This problem is then a standard problem in linear system theory where standard algorithms can be applied.

The transformation from a rational problem to a polynomial problem can be done in several different ways. In this section, two possibilities are demonstrated, where one is used if the model is given on transfer function form and the other if the model is given in state-space form. If there are no disturbances d, the problem of finding a basis to the left null-space of $M(s)$, is simplified and a method applicable in this case will also be described. Altogether, the results in this section will give us a computationally simple, efficient, and numerically stable method, to find a polynomial basis for the left null-space of $M(s)$.

Frequency Domain Solution

When the system model is given on the transfer function form (1), the transformation from the rational problem to a polynomial problem can be done by performing a right MFD on $M(s)$, i.e.

$$M(s) = \widetilde{M}_1(s)\widetilde{D}^{-1}(s) \tag{7}$$

By finding a polynomial basis for the left null-space of the *polynomial* matrix $\widetilde{M}_1(s)$, a basis is found also for the left null-space of $M(s)$. Thus the problem of finding a minimal polynomial basis to $\mathcal{N}_L(M(s))$ has been transformed into finding a minimal polynomial basis to $\mathcal{N}_L(\widetilde{M}_1(s))$.

State-Space Solution

When the system model is available in state-space form, it is here shown how the *system matrix* in state-space form can be used to find the left null-space of $M(s)$. The system matrix has been used before in the context of fault diagnosis, see e.g. (Nikoukhah 1994, Magni & Mouyon 1994).

Assume that the fault-free system is described in state-space form by,

$$\dot{x} = Ax + B_u u + B_d d \tag{8a}$$

$$y = Cx + D_u u + D_d d \tag{8b}$$

To be able to obtain a basis that is irreducible, it is required that the state x is controllable from $[u^T \ d^T]^T$.

Denote the system matrix $M_s(s)$, describing the system with disturbances as inputs:

$$M_s(s) = \begin{bmatrix} C & D_d \\ -sI + A & B_d \end{bmatrix}$$

Define a matrix P as

$$P = \begin{bmatrix} I & -D_u \\ 0 & -B_u \end{bmatrix}$$

Then the following theorem gives a direct method on how to find a minimal polynomial basis to $\mathcal{N}_L(M(s))$ via the system matrix.

Theorem 3. If the pair $\{A, [B_u \ B_d]\}$ is controllable and the rows of the polynomial matrix $V(s)$ is a minimal polynomial basis for $\mathcal{N}_L(M_s(s))$, then $W(s) = V(s)P$ is a minimal polynomial basis for $\mathcal{N}_L(M(s))$.

Before this theorem can be proven, a lemma is needed:

Lemma 4.

$$\operatorname{Dim} \mathcal{N}_L(M(s)) = \operatorname{Dim} \mathcal{N}_L(M_s(s))$$

The proof of this lemma can be found in (Nyberg 1999).

Now, return to the proof of Theorem 3:

PROOF. In the fault free case, i.e. $f = 0$, consider the following relation between the matrices $M(s)$ and $M_s(s)$:

$$P \begin{pmatrix} y \\ u \end{pmatrix} = PM(s) \begin{pmatrix} u \\ d \end{pmatrix} = M_s(s) \begin{pmatrix} x \\ d \end{pmatrix}$$

If $V(s)M_s(s) = 0$, then the above expression is zero for all x and d, which implies that $W(s)M(s) = V(s)PM(s) = 0$, i.e. $W(s) \in \mathcal{N}_L(M(s))$. It is also immediate that if $V(s)$ is polynomial, $W(s) = V(s)P$ is also polynomial.

From Lemma 4, we have that $\operatorname{Dim} \mathcal{N}_L(M_s(s)) = \operatorname{Dim} \mathcal{N}_L(M(s))$. Then since both $V(s)$ and $W(s)$ has the same number of rows, the rows of $W(s)$ must span the whole null-space $\mathcal{N}_L(M(s))$, i.e. $W(s)$ must be a basis for $\mathcal{N}_L(M(s))$.

It is clear that the following relation must hold:

$$V(s)[P \ M_s(s)] = V(s) \begin{bmatrix} I & -D_u & C & D_d \\ 0 & -B_u & -(sI - A) & B_d \end{bmatrix} = \\ = [W(s) \ 0] \tag{9}$$

Since the state x is controllable from u and d, the PBH test (Kailath 1980, p. 366) implies that the lower part of the matrix $[P \ M_s(s)]$ has full rank for all s, i.e. it is irreducible. Now assume that $W(s)$ is not irreducible. This means that there exists a s_0 and a $\gamma \neq 0$ such that $\gamma V(s_0)[P \ M_s(s_0)] = \gamma[W(s_0) \ 0] = 0$. Since $[P \ M_s(s_0)]$ has full row-rank it must hold that $\gamma V(s_0) = 0$. However, this contradicts the fact that $V(s)$ is a minimal polynomial basis. This contradiction implies that $W(s)$ must be irreducible.

Now, partition $V(s) = [V_1(s) \ V_2(s)]$ according to the partition of $M_s(s)$. Since $V(s) \in \mathcal{N}_L(M_s(s))$, it holds that

$$V_1(s)C = V_2(s)(sI - A) = sV_2(s) - V_2(s)A$$

Also, since each row-degree of $sV_2(s)$ is strictly greater than the corresponding row-degree of $V_2(s)A$, it holds that for each row i

$$\text{row-deg}_i \ sV_2(s) = \text{row-deg}_i \ V_2(s) + 1 = \text{row-deg}_i \ V_1(s)C$$

The above equation can be rearranged into the inequalities

$$\text{row-deg}_i \ V_2(s) < \text{row-deg}_i \ V_1(s)C \leq \text{row-deg}_i \ V_1(s)$$

This implies that $V_{hr} = [V_{1,hr} \ 0]$ where V_{hr} and $V_{1,hr}$ are the highest-row-degree coefficient matrices of $V(s)$ and $V_1(s)$ respectively. Since $V(s)$ is a minimal polynomial basis V_{hr} has full row rank from which it follows that $V_{1,hr}$ has full row rank.

From the definition of P it follows that

$$[W_1(s)\ W_2(s)] = [V_1(s) \quad (-V_1(s)D_u - V_2(s)B_u)] \quad (10)$$

From (10) it follows that the highest-row-degree coefficient matrix of $W(s)$ looks like $W_{hr} = [V_{1,hr}\ \star]$ where $\star$ is any constant matrix. Since $V_{1,hr}$ has full row-rank so has W_{hr}, i.e. $W(s)$ is row reduced.

Thus we have shown that $W(s)$ is an irreducible basis and row reduced, which implies that it is a minimal polynomial basis thus ending the proof.

Remark: If the realization is not controllable from $\left[u^T\ d^T\right]^T$, then it can be shown (Nyberg 1999), that the matrix $W(s) = V(s)P$ becomes a basis but not necessarily irreducible. This has the implication that all decoupling polynomial vectors $F(s)$ can *not* be parameterized as in (5).

4.1 *No Disturbance Case*

If there are no disturbances, i.e. $H(s) = 0$, the matrix $M(s)$ has a simpler structure:

$$M_{nd}(s) = \begin{bmatrix} G(s) \\ I \end{bmatrix} \quad (11)$$

A minimal polynomial basis for the left null-space of $M_{nd}(s)$ is particularly simple due to the special structure, and a minimal basis is then given directly by the following theorem:

Theorem 5. If $G(s)$ is a proper transfer matrix and $\bar{D}_G(s)$, $\bar{N}_G(s)$ form an irreducible left MFD, i.e. $\bar{N}_G(s)$ and $\bar{D}_G(s)$ are left co-prime and $G(s) = \bar{D}_G^{-1}(s)\bar{N}_G(s)$, and $\bar{D}_G(s)$ is row-reduced, then

$$N_M(s) = [\bar{D}_G(s)\ \ -\bar{N}_G(s)] \quad (12)$$

forms a minimal polynomial basis for the left null-space of the matrix $M_{nd}(s)$, i.e. $\mathcal{N}_L(M_{nd}(s))$.

PROOF. It is immediate to evaluate

$$[\bar{D}_G(s)\ \ -\bar{N}_G(s)] \begin{bmatrix} G(s) \\ I \end{bmatrix} = 0$$

Also, the dimension of the left null-space of $M_{nd}(s)$ has dimension m, i.e. the number of measurements, which equals the number of rows in $N_M(s)$. Since $\bar{D}_G(s)$ and $\bar{N}_G(s)$ is co-prime, $N_M(s)$ will be irreducible. Let D_{hr} and N_{hr} be the highest row-degree coefficient matrices for $\bar{D}_G(s)$ and $\bar{N}_G(s)$ where D_{hr} will be of full rank since $\bar{D}_G(s)$ is row-reduced.

Since the transfer function $G(s)$ is proper, i.e. the row degrees of $\bar{N}_G(s)$ is less or equal to the row degrees of $\bar{D}_G(s)$. A high-degree coefficient decomposition of the basis $N_M(s)$ will look like

$$[\bar{D}_G(s)\ \ -\bar{N}_M(s)] = S_D(s)[D_{hr}\ \star] + \tilde{L}(s)$$

where $\star$ is any matrix. Since D_{hr} is full rank, so is $[D_{hr}\ \star]$, which gives that the basis is row-reduced which ends the proof.

*Remark 1:*Note that not just any irreducible left MFD will suffice. According to Theorem 5, matrix $\bar{D}_G(s)$ need to be row-reduced. An algorithm that gives such an MFD is (Strijbos 1996) and is implemented in *The Polynomial Toolbox 2.0 for Matlab 5* (1998).

The *order* of the minimal polynomial basis for $\mathcal{N}_L(M_{nd}(s))$ is given by the following theorem:

Theorem 6. The set of observability indices of a transfer function $G(s)$ is equal to the set of row degrees of $\bar{D}_G(s)$ in any row-reduced irreducible left MFD $G(s) = \bar{D}_G^{-1}(s)\bar{N}_G(s)$.

A proof of the dual problem, controllability indices, can be found in (Chen 1984, p. 284).

Thus, a minimal polynomial basis for the left null-space of matrix $M_{nd}(s)$ is given by a left MFD of $G(s)$ and the order of the basis is the sum of the observability indices of $G(s)$.

Remark 2: Note that, in the general case, the observability indices of the pair $\{A, C\}$ does not give the row-degrees of a minimal polynomial basis for $\mathcal{N}_L(M(s))$. However, the minimal observability index of $\{A, C\}$ do give a lower bound on the minimal row-degree of the basis (Frisk 2000).

Remark 3: The result (12) implies that finding the left null-space of the rational transfer matrix (3), in the general case with disturbances included, can be reduced to finding the left null-space of the rational matrix

$$\widetilde{M}_2(s) = \bar{D}_G(s)H(s) \quad (13)$$

In other words, this is an alternative to the use of the matrix $\widetilde{M}_1(s)$ in (7). This view closely connects with the so called frequency domain methods, which are further examined in Section 6.

4.2 *Finding a Minimal Polynomial Basis for the null-space of a General Polynomial Matrix*

For the general case, with disturbances included, the only remaining problem is how to find a minimal polynomial basis to a general polynomial matrix. This is a well-known problem in the general literature on linear systems. Several algorithms exists but when numerical performance is considered, a specific algorithm based on the *polynomial echelon form* (Kailath 1980) has been proven to be both fast and numerically stable. Such an algorithm is implemented in the command `null` in *The Polynomial Toolbox 2.0 for Matlab 5* (1998).

5. BOUNDS ON MAXIMUM AND MINIMUM ROW-DEGREE OF THE BASIS

This section derives upper limits on the maximum and minimum row-degree of a matrix, whose rows form a minimal polynomial basis for the left null-space of the matrix (3). The notation n is used to denote the number of states in a given state-space representation. The notation n_x will be used to denote the dimension of the state controllable from $\left[u^T\, d^T\right]^T$.

5.1 *Upper Bound for the Maximum Row-Degree of the Basis*

Theorem 7. (Nyberg,1999). A matrix whose rows form a minimal polynomial basis for $\mathcal{N}_L(M(s))$ has all row-degrees $\leq n_x$, where n_x is the number of states controllable from $\left[u^T\, d^T\right]^T$.

Before Theorem 7 can be proven, a few lemmas are needed.

Lemma 8. Let $P(s)$ be a matrix with maximum row-degree 1. Then the maximum row-degree of a minimal polynomial basis for $\mathcal{N}_L(P(s))$ is less or equal to $\operatorname{Rank} P(s)$.

Lemma 9. The row-degrees of a minimal polynomial basis for $\mathcal{N}_L(M(s))$ is equal to the row-degrees of a minimal polynomial basis for $\mathcal{N}_L(M_s(s))$, where $M_s(s)$ is a system matrix with the pair $\{A, [B_u\; B_d]\}$ controllable.

Remark: Lemma 9 implies that the row-degrees of $N_M(s)$ equals the left Kronecker indices of the matrix pencil $M_s(s)$. For proof of Lemma 8, see e.g. (Nyberg 1999) and for proof of Lemma 9, see e.g. (Frisk 2000).

Lemma 10. (Predictable-degree Property of row-reduced matrices). Let $D(s)$ be a polynomial matrix of full row-rank, and for any polynomial vector $p(s)$, let

$$q(s) = p(s)D(s)$$

Then, $D(s)$ is row-reduced if and only if

$$\deg q(s) = \max_{i:p_i(s)\not\equiv 0} [\deg p_i(s) + \mu_i]$$

where $p_i(s)$ is the i:th entry of $p(s)$ and μ_i is the degree of the i:th row of $D(s)$.

Lemma 11. Let the rows of $F(s)$ form a minimal polynomial basis for a rational vectorspace $\mathcal{F}$. Denote the row-degrees of $F(s)$ with $\mu_1 \leq \cdots \leq \mu_\alpha$. Then it holds that $\mu_i \leq m_i, i = 1, \ldots, \alpha$ where m_i is the row-degrees of any polynomial basis for $\mathcal{F}$.

PROOF. Let $P(s)$ be a polynomial basis for $\mathcal{F}$ with row-degrees m_i. Let the rows in $P(s)$ be ordered such that $m_1 \leq \cdots \leq m_\alpha$.

The theorem is proved by contradiction. Assume that $\mu_1 \leq m_1, \ldots, \mu_{i-1} \leq m_{i-1}$ but $\mu_i > m_i$. Since $F(s)$ is an irreducible basis, it holds that

$$p_j(s) = \sum_{l=1}^{\alpha} f_l(s) q_l(s) \quad j = 1, \ldots, i \tag{14}$$

where $q_l(s)$ is polynomials.

If $i = 1$, then $\deg p_1(s) < \mu_j \quad j = 1, \ldots, \alpha$, i.e. according to the predictable degree property $p_1(s)$ can not be a linear combination of the rows in the row reduced matrix $F(s)$. However, this contradicts (14).

If $i > 1$, according to the assumption, the following relations hold:

$$\deg p_j(s) \leq m_i < \mu_i \quad j = 1, \ldots, i$$

According to the predictable degree property it must hold that in (14), $q_l(s) \equiv 0, l = i, \ldots, \alpha$. Thus, the upper summation limit can at maximum be $i-1$, i.e. equation (14) can be rewritten as:

$$p_j(s) = \sum_{l=1}^{i-1} f_l(s) q_l(s) \quad j = 1, \ldots, i$$

This contradicts the linear independence of the $p_1(s), \ldots, p_i(s)$ polynomial row vectors since they are spanned by $f_1(s), \ldots, f_{i-1}(s)$ ending the proof.

Now return to the proof of Theorem 7:

PROOF. Let n_x be the order of a minimal state-space realization of (8), controllable from $\left[u^T\, d^T\right]^T$. Let $M_s(s)$ be the corresponding system matrix, i.e.

$$M_s(s) = \begin{bmatrix} C & D_d \\ -(sI - A) & B_d \end{bmatrix}$$

and let the rows of N_{DB} be a basis for the left null-space of $[D_d^T\; B_d^T]^T$. Then we have that

$$N_{DB} M_s(s) = \left[N_{DB} \begin{bmatrix} C \\ -(sI - A) \end{bmatrix}, 0\right] \tag{15}$$

The left part of the matrix (15) has rank $\leq n_x$. From Lemma 8 we know that a minimal polynomial basis for (15) has row degrees less or equal to n_x. Let the rows of a matrix $Q(s)$ form such a basis.

The matrix $Q(s)N_{DB}$ forms a polynomial basis for $\mathcal{N}_L(M_s(s))$ and since $Q(s)$ has row degrees less or equal to n_x, the row degrees of the basis $Q(s)N_{DB}$ is also less or equal to n_x. Thus, according to Lemma 11, a *minimal* polynomial basis for $\mathcal{N}_L(M_s(s))$ has lower or equal row-degrees than the polynomial basis $Q(s)N_{DB}$.

Since a minimal polynomial basis for $\mathcal{N}_L(M_s(s))$ has maximum row-degree $\leq n_x$, Lemma 9 implies that also a minimal polynomial basis for $\mathcal{N}_L(M(s))$ has maximum row-degree $\leq n_x$, ending the proof.

The result of Theorem 7 is important for several reasons, the residual generators obtained directly from the vectors of the minimal basis, are in one sense the only ones needed. All other are filtered versions (i.e. linear combinations) of these residual generators. With this argument, Theorem 7 shows that we do not need to consider residual generators of orders greater than n_x. Also, in the Chow-Willsky scheme, see Section 6.1, the maximum order ρ needs to be specified. By letting $\rho = n_x$, it is guaranteed that all vectors in the minimal polynomial basis can be generated.

Remark 1 It is also possible to show Theorem 7 by using results from (Henrion & Sebek 1999, Lemma 1).

Remark 2 Related problems have been investigated in (Chow & Willsky 1984) and (Gertler, Fang & Luo 1990). In (Chow & Willsky 1984), it was shown that, in the no-disturbance case, there exist a parity function of order $\leq n$. In (Gertler et al. 1990), it was shown that for a restricted class of disturbances, there exist a parity function of order $\leq n$. However the result of Theorem 7 is stronger since it includes *arbitrary* disturbances and shows that there exist a *basis* in which the maximum row-degree is $\leq n_x$.

5.2 *Upper Bound for the Minimal Row-Degree of the Basis*

Theorem 12. (Frisk,2000). An upper bound for the minimal row-degree ρ_{min} of a basis for $\mathcal{N}_L(M(s))$ is given by

$$\rho_{\text{min}} \leq \lfloor \frac{n_x + \tilde{k}_d}{m - \tilde{k}_d} \rfloor$$

where

$$\tilde{k}_d = \text{Rank} \begin{pmatrix} B_d \\ D_d \end{pmatrix}$$

is the number of linearly independent disturbances.

Before Theorem 12 can be proven, some more results are needed. If $\tilde{k}_d < k_d$, i.e. there exists linear dependencies between disturbances, rewrite the system description with a new set of $\tilde{k}_d$ linearly independent disturbances. That is, find $\widetilde{B}_d$ and $\widetilde{D}_d$ with dimensions $n_x \times \tilde{k}_d$ and $m \times \tilde{k}_d$ respectively such that

$$\text{Im} \begin{pmatrix} B_d \\ D_d \end{pmatrix} = \text{Im} \begin{pmatrix} \widetilde{B}_d \\ \widetilde{D}_d \end{pmatrix}$$

Then, denote

$$\widetilde{M}_\rho = \underbrace{\left.\begin{bmatrix} Q & R & & \\ & Q & R & \\ & & \ddots & \\ & & Q & R \end{bmatrix}\right\}}_{(\rho+2)(n_x+\tilde{k}_d)} (\rho+1)(m+n_x)$$

where $M_s(s) = Q + sR$. Then, the following lemma can be stated:

Lemma 13. (Karcanias & Kalogeropoulos,1988). The space $\mathcal{N}_L(M_s(s))$ contains a ρ-degree polynomial vector if and only if $\widetilde{M}_\rho$ does not have full row rank.

Now, return to the proof of Theorem 12.

PROOF. Using Lemma 9 and 13 it is clear that a ρ-degree polynomial vector is in $\mathcal{N}_L(M(s))$ if and only if $\widetilde{M}_\rho$ does not have full row rank. A sufficient condition for $\widetilde{M}_\rho$ not to have full row-rank is that the number of rows is larger than the number of columns, i.e.

$$(\rho + 1)(m + n_x) > (\rho + 2)(n_x + \tilde{k}_d)$$

Straightforward manipulations of the inequality results in

$$\rho > \frac{n_x + \tilde{k}_d}{m - \tilde{k}_d} - 1$$

Note that the inequality $m > \tilde{k}_d$ always holds if a residual generator exists which can be seen directly in (4). Therefore, the smallest integer ρ that fulfills the inequality is $\lfloor \frac{n_x+\tilde{k}_d}{m-\tilde{k}_d} \rfloor$ which completes the proof.

Remark: A similar result without disturbance decoupling, i.e. when $\tilde{k}_d = 0$, can be found in (Mironovskii 1980).

6. RELATION TO OTHER RESIDUAL GENERATOR DESIGN METHODS

This section discusses the relation between the minimal polynomial basis approach and two other design methods for linear residual generation.

6.1 *The Chow-Willsky Scheme*

A method for constructing linear residual generators was presented in (Chow & Willsky 1984). This method is usually referred to as the Chow-Willsky scheme (or the *parity-space approach*). Originally it was only described for the no-disturbance case. However, in (Frank 1990), the method was generalized to also include decoupling of disturbances.

Since the Chow-Willsky scheme is well known, only a short description is given here. Designing residual generators with the Chow-Willsky scheme comes down to finding a vector w in the left null-space of a constant matrix $[\mathcal{O}_\rho \; H_\rho]$, where

$$\mathcal{O}_\rho = [C^T \; A^T C^T \; \ldots \; A^{\rho T} C^T]^T$$

and H_ρ is a lower triangular Toeplitz matrix describing the propagation of the disturbances through the system. The constant ρ is determined by the user. Let w be a row vector in the left null-space of $[\mathcal{O}_\rho \; H_\rho]$ and form the polynomial vector $F_{CW}(s)$ as

$$F_{CW}(s) = w\,[\Psi_y(s) \quad -Q_\rho \Psi_u(s)]$$

where

$$\Psi_y(s) = [I_m \; sI_m \; \ldots \; s^\rho I_m]^T \quad \Psi_u(s) = [I_k \; sI_k \; \ldots \; s^\rho I_k]^T$$

and Q_ρ is a lower triangular Toeplitz matrix describing the propagation of the inputs through the system. Compare $F_{CW}(s)$ with $F(s)$ in 5. It can be shown that $F_{CW}(s)$ always belongs to the left null-space of $M(s)$. This means that the Chow-Willsky scheme is a method for finding polynomial vectors in the left null-space of $M(s)$. In accordance with the minimal polynomial basis approach, a realizable residual generator can be achieved by adding poles as described by the formula (6).

It has been shown in (Nyberg & Nielsen 2000) that for some systems, the Chow-Willsky scheme can not generate all possible residual generators. This is the case when there are dynamics controllable from the faults but not from inputs or disturbances. However, by imposing the same requirement as in Theorem 3, about controllability from $[B_u \; B_d]$, the Chow-Willsky scheme can be modified so that it can generate all possible residual generators.

The row degree of the vector $F_{CW}(s)$ will in most cases be equal to ρ. Therefore, the Chow-Willsky scheme does not *automatically* find residual generators of minimal order. By iteratively increasing ρ it is straightforward to find a minimal order parity relation, however such an approach is *not* able to produce a basis such as $N_M(s)$ and to ensure that all parity relations is found, $\rho = n$ is needed.

Commonly, the Chow-Willsky scheme is formulated in the following way. First a basis for the left null-space of $[\mathcal{O}_\rho \; H_\rho]$ is found. Let the rows of a matrix W define this basis. Then the vector w, used to form the polynomial vector $F_{CW}(s)$, is taken as one of the rows in W or possibly a linear combination of several rows. It is worth to point out that the first step to find the basis W, does not result in a polynomial basis to the left null-space of $M(s)$, i.e. $W\,[\Psi_y(s) \;\; -Q_\rho \Psi_u(s)]$ is not a basis for $\mathcal{N}_L(M(s))$. This is easy to realize since the number of rows in W is in general much larger than the dimension of the null space.

In conclusion, the Chow-Willsky scheme does not give residual generators of minimal order and also, a basis is not obtained. However, by using several modifications to the original algorithm, it is possible to obtain a modified Chow-Willsky scheme which will produce a minimal polynomial basis for the left null-space of $M(s)$. The modified Chow-Willsky scheme and the equivalence with the minimal polynomial basis approach is described in (Nyberg 1999).

Although the Chow-Willsky scheme can be made algebraically equivalent to the minimal polynomial basis approach, the numerical properties are still not as good. The reason is that, for anything but small ρ, the matrix $[\mathcal{O}_\rho \; H_\rho]$ will include high powers of A. It is likely that this results in that $[\mathcal{O}_\rho \; H_\rho]$ becomes ill-conditioned. Thus to find the left null-space of $[\mathcal{O}_\rho \; H_\rho]$ can imply severe numerical problems. When the minimal polynomial basis approach is used, and a basis for the left null-space of the system matrix is derived using the polynomial echelon-form algorithm, these problems of high power of A, or any other term, will not arise.

6.2 *Frequency Domain Approaches*

A number of design methods described in literature are called *frequency domain methods* where the residual generators are designed with the help of different transfer matrix factorization techniques. Examples are (Frank & Ding 1994) for the general case with disturbances and (Ding & Frank 1990, Viswanadham, Taylor & Luce 1987) in the non-disturbance case. The methods can be summarized as methods where the residual generator is parameterized as

$$r = R(s)[\tilde{D}(s) \; -\tilde{N}(s)] \begin{pmatrix} y \\ u \end{pmatrix} = \\ = R(s)(\tilde{D}(s)y - \tilde{N}(s)u) \quad (16)$$

where $\tilde{D}(s)$ and $\tilde{N}(s)$ form a left co-prime factorization of $G(s)$ over $\mathcal{RH}_\infty$. Note the close relationship with Equation (12) where the factorization is performed over polynomial matrices instead of over $\mathcal{RH}_\infty$.

Inserting (1) into Equation (16) and as before assuming $f = 0$, gives

$$r = R(s)\tilde{D}(s)H(s)d$$

Therefore, to achieve disturbance decoupling, the parameterization transfer matrix $R(s)$, must belong to the left null-space of $\tilde{D}(s)H(s)$, i.e.

$$R(s)\tilde{D}(s)H(s) = 0$$

Here, note the close connection with $\widetilde{M}_2(s)$ in (13). This solution however does not generally generate a residual generator with minimal order. In (Ding & Frank 1990) and (Frank & Ding 1994), the co-prime factorization is performed via a minimal state-space realization of the complete system, including the disturbances as in Equation (8). This results in $\tilde{D}(s)$ and $\tilde{N}(s)$ of a degree that, in the general case, is larger than the lowest possible order of a disturbance decoupling residual generator. Thus, to find a lowest order basis that spans all residual generators $Q(s) = R(s)[\tilde{D}(s) \;\; -\tilde{N}(s)]$, extra care is required since "excess" states need to be canceled out.

7. DESIGN EXAMPLES

7.1 *Design Example 1: Aircraft Dynamics*

The model used in this example is taken from (Maciejowski 1989) and represents a linearized model of vertical-plane dynamics of an aircraft. The model has three inputs, three outputs. Numerical values for model equations can be found in (Maciejowski 1989). Suppose the faults of interest are sensor-faults (denoted f_1, f_2, and f_3), and actuator-faults (denoted f_4, f_5, and f_6) and assume additive fault models. The total model, including the faults then becomes:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = G(s)\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} f_4 \\ f_5 \\ f_6 \end{bmatrix}\right) + \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

where $G(s)$ is the nominal system model.

The design example is intended to illustrate the design procedure and also illustrate how available design freedom can be utilized. The goal is to design a residual generator $Q(s)$ that decouples the fault in the elevator angle actuator, i.e. f_6 is a non-monitored fault. Therefore the matrix $H(s)$ in (1), corresponding to non-monitored faults and disturbances, becomes equal to the third column of $G(s)$. The matrix $L(s)$ corresponds to the monitored faults and therefore $L(s)$ becomes $[I_3\ g_1(s)\ g_2(s)]$, where $g_i(s)$ denotes the i:th column of $G(s)$.

According to formula (4), the dimension of the null-space $\mathcal{N}_L(M(s))$ is 2, i.e. there exists exactly two linearly independent polynomial row-vectors that decouples f_6. Theorem 12 gives an upper bound on the minimum degree residual generator of $\lfloor \frac{5+1}{3-1} \rfloor = 3$. Calculations using *The Polynomial Toolbox 2.0 for Matlab 5* (1998) and Theorem 3 give:

$$N_M(s) = \begin{bmatrix} 0.0705s & s + 0.0538 & \dots \\ 22.7459s^2 + 14.5884s & -6.6653 & \dots \end{bmatrix}$$
$$\begin{matrix} 0.091394 & 0.12 & -1 & 0 \\ s^2 - 0.93678s - 16.5141 & 31.4058 & 0 & 0 \end{matrix}\Bigg] \quad (17)$$

The row-degrees of the basis is 1 and 2, i.e. it is a basis of order 3.

Forming the Residual Generator

From the basis (17) it is clear that a proper filter of least order, which decouples f_6, is a first order filter corresponding to the first row in the basis. A realizable residual generator can be formed by setting ϕ in (5) to $\phi = [1\ 0]$ and $c(s)$ to $c(s) = 1 + s$ which results in:

$$Q(s) = \frac{1}{1+s}\begin{bmatrix} 0.0705s & s + 0.0538 & 0.091394 & 0.12 & -1 & 0 \end{bmatrix} \quad (18)$$

Let $G_d(s) = Q(s)\begin{pmatrix} G(s) & H(s) \\ I & 0 \end{pmatrix}$, which should be zero if infinite precision arithmetics were used. Calculating the size of $G_d(s)$ using the infinity norm gives $\|G_d(s)\|_\infty \approx -220$ dB which is close to machine precision, i.e. control signals and the decoupled fault has no significant influence on the residual.

Figure 1 show how the monitored faults influence the residual and the leftmost plot shows that the DC-gain from f_1 to the residual is 0. Therefore, f_1 is difficult to detect since the effect in the residual of a constant fault disappears. By using *detectability criterions*, given in (Nyberg 1999), it can be shown that it is impossible to construct a residual generator in which the DC-gain from f_1 to the residual becomes non-zero.

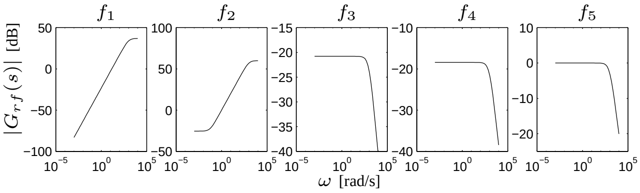

Fig. 1. Magnitude bode plots for the monitored faults to the residual.

In the example, the dimension of the null-space when decoupling f_6 was 2 as seen in (17). This indicates that there exists additional freedom that can e.g. be used to attenuate noise or to decouple more than one fault in each residual to facilitate multiple-fault isolation. Further investigations on this example can be found in (Frisk & Nyberg 1999).

7.2 *Design Example 2: A Turbo-Jet Engine from Volvo Aero Corporation*

This second example is included to illustrate numerical and other properties of the algorithm described. Here, a short discussion is included, a more detailed discussion on this example can be found in (Frisk 1998).

A model of a jet-engine developed by Volvo Aero Corporation, Trollhättan, Sweden, is used in this example. A high-order non-linear model of the engine is used for analysis and control design. This model can also be used for diagnosis purposes. The model was linearized in a working point and the resulting model, after that non-controllable and non-observable modes are eliminated, is a 26:th order model. The model used includes 8 sensors and 4 actuators.

The model is numerically stiff due to modeling of fast dynamics, such as thermodynamics in small control volumes, and slow dynamics such as heating phenomena of metal. The largest time-constant in the model is about 10^5 times larger than the smallest time constant. This, together with the high-order, makes the model numerically sensitive which demands good numerical properties of the design algorithm.

In the design example, faults in sensors and actuators are considered. A residual that indicates a sensor failure is to be designed, i.e. all actuator faults are to be decoupled. Using Theorem 12 it is clear that there exists residual generators with degree less than or equal to $\lfloor \frac{26+4}{8-4} \rfloor = 7$ which is significantly less than system order. Worth noting is how this limit depends on n_d. If a residual were to be designed that decoupled only one fault, i.e. $n_d = 1$, then the upper bound on the minimum degree residual generator would be as low as 3.

Experiments in Matlab shows that the algorithm performs well on the full model resulting in a 4:th order filter giving $\approx$ 0 dB gain from the sensor faults to the residual while the actuators has $\approx$ -200 dB gain to the residual, i.e. the decoupling succeeded. To illustrate the numerical difficulties in this example, a design is also performed with the Chow-Willsky scheme (Chow & Willsky 1984). Performing the same design with the *basic* Chow-Willsky design method, i.e. exactly as outlined in Section 6.1 with $\rho = n$, on a balanced realization of the model result in an infeasible design. The design fails due to severe numerical difficulties during the design. The main point of this comparison is *not* to make any statement whether it is possible to perform a feasible design on this example with the Chow-Willsky method or not. It is merely used to illustrate that this example is numerically difficult and numerical concern is important.

It is also worth noting that, a design method not considering the order of the resulting residual generator easily results in a residual generator of the same order as the process model, here 26. However, with the minimal polynomial basis approach, a 4:th order residual generator was found which shows how the minimality property here results in a filter with substantially less order than the order of the design model. This order gain, i.e. reduced order of the residual generator, can be substantial, especially when using detailed, high-order design models.

Another possibility to decrease filter order is to utilize some order reduction technique on the high-order model, and then design the residual generator. The minimal polynomial approach has advantages compared to such an approach, mainly due to that it avoids an unnecessary decrease in model accuracy. In the jet-engine example, it was not possible to reduce the model order to 4, design a residual generator, and end up with a filter with the same performance as the resulting filter from the minimal polynomial basis approach.

8. CONCLUSIONS

Design of residual generators to achieve perfect decoupling in linear systems is considered. The goal has been to develop a design method where especially three issues have been addressed: (1) the method is able to generate *all* possible residual generators, (2) explicitly gives the solutions with minimal order, and (3) has good numerical properties.

The residual generator design problem is formulated with standard notions from linear algebra and linear systems theory such as polynomial bases for rational vector spaces and it is shown that the design problem can be seen as the problem of finding *polynomial* vectors in the left null-space of a rational matrix $M(s)$. Within this framework, the completeness of solution, i.e. issue (1) above, and minimality, i.e. issue (2), are naturally handled by the concept of *minimal polynomial bases*. Finding a minimal polynomial basis for a null-space is a well-known problem and there exists computationally simple, efficient, and numerically stable algorithms, i.e. issue (3), to generate the bases.

Simple bounds on the row-degrees of such a minimal polynomial basis are derived and it is also shown how these degrees are closely related to the order of the residual generators. These bounds can help the designer to estimate complexity of the diagnosis system.

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