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On observing nonlinear descriptor systems

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Abstract

We study the problem of observing the state of continuous nonlinear descriptor systems in quasilinear form and present a method to construct a state observer. Our approach is based on rewriting the descriptor system as an equivalent system of (explicit) differential equations on a restricted manifold. Finally, the restrictions are replaced by measurement equations. Thus, an observer for the descriptor system can be constructed by common state space techniques for explicit systems. © 1997 Elsevier Science B.V.

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1. Introduction

Mathematical models of electric circuits, of interconnected large-scale systems, of mechanical systems with holonomic or nonholonomic constraints or of robotic systems with kinematical constraints often result in differential algebraic equations, also known as descriptor systems. The problem of controlling and observing linear descriptor systems has been intensively studied during the past years. See [1] and references therein. Controlling and stabilizing a class of nonlinear descriptor systems which is suitable to describe robotic models are quite well understood, see [10,6,5]. Literature concerned with the control aspect of more general nonlinear descriptor systems is sparse.

Here we will address the problem of observing the state of a nonlinear descriptor system. Commonly state observation is carried out by building a parallel system where the output of the original system is used as an additional control input. The aim is to steer the state of the parallel system asymptotically towards the state of the original system (see e.g. [7]). Since nonlinear descriptor systems may be algebraically incomplete [9] it is not advisable to transfer this concept directly. Algebraic incompleteness causes difficulties in solving nonlinear descriptor systems numerically and thus entails problems implementing the parallel system.

Our attempt is as follows: Based on [9, 8], we propose a method to transform a regular descriptor system into a corresponding explicit differential equation on a reduced manifold. In a second step, we replace the restrictions that determine this manifold by additional measurements. The resulting system will be used as parallel system for the state observation, where we can carry on with well-known techniques to design state observers.

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2. Preliminaries

Let $\mathcal{M} \subset \mathbb{R}^n$ denote an embedded submanifold, $T\mathcal{M}$ the tangent bundle and $T_x\mathcal{M}$ the tangent space at $\mathcal{M}$ in $x \in \mathcal{M}$.

We focus on nonlinear descriptor systems in quasi-linear form on $\mathcal{M}$

$$A(x)\dot{x} = f(x), \tag{1a}$$

$$y = h(x), \tag{1b}$$

$x \in \mathcal{M}$, $A \in C^k(\mathbb{R}^n, \mathbb{R}^{n\times n})$, $f \in C^k(\mathbb{R}^n, \mathbb{R}^n)$ and $h \in C^k(\mathbb{R}^n, \mathbb{R}^m)$ $(k > n)$. We assume $f(0) = 0$ and $\operatorname{rank}(A(x)) = q < n$ for all $x \in \mathbb{R}^n$. $(\varphi \in C^1(I, \mathcal{M}), I)$, I an open interval (not necessarily bounded), is called a *solution of the quasilinear descriptor system* (1a) on $\mathcal{M}$ iff $A(\varphi(t))\dot{\varphi}(t) = f(\varphi(t))$ for all $t \in I$.

Following Reich [8] we define

Definition 1. Eq. (1a) is called a *regular system* iff there exists an embedded submanifold $\mathcal{R} \subset \mathcal{M}$ and a vector field $v \in C(\mathcal{R}, T\mathcal{R})$ such that each solution of the differential equation $\dot{x} = v(x)$, $x \in \mathcal{R}$, is a solution of (1a) and vice versa.

$\mathcal{R}$ is the *configuration space* of (1a) and v is a *corresponding vector field* of (1a).

Two descriptor systems (1a) and

$$A^{\#}(w)\dot{w} = f^{\#}(w),$$

$$w \in \mathcal{M}^{\#} \subset \mathbb{R}^n, \quad \dim(\mathcal{M}^{\#}) \leqslant \dim(\mathcal{M}) \tag{2}$$

are *equivalent* iff there exists a diffeomorphism $u : \mathbb{R}^n \to \mathbb{R}^n$ and a mapping $S : \mathbb{R}^n \to \mathbb{R}^{n\times n}_{\mathrm{reg}}$ such that for any $x \in \mathcal{M}$, $X \in T_x\mathcal{M}$ with $u(x) \in \mathcal{M}^{\#}$ and $X^{\#} := Du(x)X \in T_{u(x)}\mathcal{M}^{\#}$,

1. $f(x) = S(x)f^{\#}(u(x))$ and $A(x)X = S(x)A^{\#}(u(x))(Du(x))^{-1}X^{\#}$ and
2. $(u \circ \varphi, I)$ is a solution of (2) iff (φ, I) is a solution of (1a).

If Definition 1 holds locally, we will use the terms locally as well.

For a regular system (1a) the configuration space $\mathcal{R}$ and v restricted to $\mathcal{R}$ are unique [8]. Thus, a solution of (1a) passing through $x^0 \in \mathcal{R}$ at time $t = 0$ is unique, too, and will be denoted by $(x(\cdot\,; 0, x^0), I_{x^0})$.

Finally, the following lemma taken from [2] is needed:

Lemma 1. *Let* $S \subset \mathbb{R}^n$ *be open,* $1 \leqslant q < n$ *and* $\gamma \in C^r(S, \mathbb{R}^q)$.

1. $\mathcal{R}(\gamma, S) := \{x \in S \mid \operatorname{rank}(D\gamma(x)) = q\}$ *is an open set in* $\mathbb{R}^n$.
2. *If there exists* $x \in \mathcal{R}(\gamma, S)$ *with* $\gamma(x) = 0$ *then* $\mathcal{M}(\gamma, S) := \{x \in \mathcal{R}(\gamma, S) \mid \gamma(x) = 0\}$ *is a nonempty embedded* C^r*-submanifold in* $\mathbb{R}^n$ *with dimension* $n - q$.

3. Index and corresponding system

Our first aim is to determine the configuration space and a corresponding vector field for a regular descriptor system (1a). For this purpose we consider a p-dimensional embedded submanifold $\mathcal{M}$ $(1 \leqslant p \leqslant n)$ as being defined through $\gamma \in C^k(\mathbb{R}^n, \mathbb{R}^{n-p})$ by $\mathcal{M} := \{x \in \mathbb{R}^n \mid \gamma(x) = 0\}$. We further assume $0 \in \mathcal{M}$ and $\operatorname{rank}(D\gamma(x)) = n - p$ for all $x \in \mathbb{R}^n$. In the following we will understand every statement as locally near $x = 0$.

Lemma 2. *Let* $A \in C^k(\mathbb{R}^n, \mathbb{R}^{n\times n})$ *with* $\operatorname{rank}(A(x)) = q < n$ *for all* $x \in \mathbb{R}^n$. *Then for all* $x^0 \in \mathbb{R}^n$ *there exists* $U(x^0) \subset \mathbb{R}^n$, $P \in C^k(U(x^0), \mathbb{R}^{n\times n}_{\mathrm{reg}})$, *and* $A_1 \in C^k(U(x^0), \mathbb{R}^{q\times n})$ *such that*

$$P(x)A(x) = \begin{pmatrix} A_1(x) \\ 0 \end{pmatrix}, \quad x \in U(x^0). \tag{3}$$

For $q = n - 1$ the transformation matrix P can be chosen globally C^k [12]. We focus on $U = U(x^0)$ and understand $\mathcal{M}$ as $\mathcal{M} \cap U$ whenever necessary.

Multiplying (1a) from the left by $P(\cdot)$ according to Lemma 2 yields the system

$$\begin{pmatrix} A_1(x) \\ 0 \end{pmatrix} \dot{x} = \begin{pmatrix} f_{1,1}(x) \\ f_{2,1}(x) \end{pmatrix}, \quad x \in \mathcal{M}, \tag{4}$$

with

$$P(x)f(x) = \begin{pmatrix} f_{1,1}(x) \\ f_{2,1}(x) \end{pmatrix},$$

$f_{1,1} \in C^k(U, \mathbb{R}^q)$, $f_{2,1} \in C^k(U, \mathbb{R}^{n-q})$.

Eq. (4) is locally equivalent to (1a). Let

$$\operatorname{rank}\begin{pmatrix} D\gamma(x) \\ Df_{2,1}(x) \end{pmatrix} = \operatorname{rank}(D\gamma(x)) + \operatorname{rank}(Df_{2,1}(x))$$

$$= n - p + n - q, \quad x \in \mathbb{R}^n.$$

Then by Lemma 1 and the assumption $f(0)=0$, the set $\mathcal{M}_1 := \{x \in \mathcal{M} \mid f_{2,1}(x)=0\} = \{x \in \mathbb{R}^n \mid \gamma(x)=0, f_{2,1}(x)=0\}$ locally defines a nonempty $p-(n-q)$-dimensional C^n-submanifold in $\mathbb{R}^n$.

Any solution (φ, I) of (1a) has to fulfill $\varphi(t) \in \mathcal{M}_1$, $(t \in I)$. If, on the other hand, every $x^0 \in \mathcal{M}_1$ defines a solution $x(\cdot\,; 0, x^0)$ of (1a), then the algebraic restriction $f_{2,1}(x)=0$ uniquely determines the set of suitable initial conditions for (1a). Thus $\mathcal{R} = \mathcal{M}_1$.

System (1a) is called *algebraically complete* iff $\mathcal{M}_1 = \mathcal{R}$.

We are mainly interested in systems which are not algebraically complete. Our aim is to reduce the manifold $\mathcal{M}$ on which the descriptor system (1a) is defined and simultaneously enlarge the rank of the matrix A on the left-hand side until the descriptor system can be related to an explicit system of differential equations on the configuration space $\mathcal{R}$.

Consider the transformed system (4) and take the derivative of $f_{2,1}$ with respect to time. Then (4) yields the system

$$\begin{pmatrix} A_1(x) \\ Df_{2,1}(x) \end{pmatrix} \dot{x} = \begin{pmatrix} f_{1,1}(x) \\ 0 \end{pmatrix}, \quad x \in \mathcal{M}_1. \tag{5}$$

Eq. (5) is called a *reduced system* assigned to (1a). Since for $x \in \mathcal{M}_1$, $f_{2,1}(x)=0$ and $Df_{2,1}(x)$ is normal to $T_x\mathcal{M}_1$, (5) is locally equivalent to (1a) and thus (locally) regular.

Definition 2. Eq. (1a) is called *system of index* 0 iff $\operatorname{rank}(A(x))=n$ for all $x \in \mathcal{M}$. Eq. (1a) is called *system of index* $i \in \mathbb{N}$ iff

1. $\operatorname{rank}(A(x)) = q < n$ for all $x \in \mathcal{M}$,
2. $$\operatorname{rank}\begin{pmatrix} D\gamma(x) \\ Df_{2,1}(x) \end{pmatrix} = \operatorname{rank}(D\gamma(x)) + n - q$$
   for all $x \in \mathcal{M}$,
3. the reduced system (5) is of index $i-1$.

An index reduction automatically terminates after at most n steps. Thus, a system with index i naturally reveals $0 \leqslant i \leqslant n$.

In the following we assume that (1a) is a system of index i and that

$$A_i(x)\dot{x} = f_i(x), \quad x \in \mathcal{M}_i, \tag{6}$$

$A_i \in C^{k-i}(U, \mathbb{R}^{n\times n})$, $f_i \in C^{k-i}(U, \mathbb{R}^n)$, is a system obtained from (1a) by i index reductions. Then by definition of i, $A_i(x)$ is nonsingular for all $x \in U$. Inverting A_i yields the system

$$\begin{aligned} \dot{x} &= A_i(x)^{-1} f_i(x), \\ x \in \mathcal{M}_i &= \{x \in \mathcal{M} \mid f_{2,1}(x)=0, \ldots, f_{2,i}(x)=0\} \\ &=: \tilde{f}(x) = \{x \in \mathbb{R}^n \mid \gamma(x)=0, f_{2,1}(x) \\ &\qquad = 0, \ldots, f_{2,i}(x)=0\}. \end{aligned} \tag{7}$$

Eq. (7) is called a *differential equation corresponding to* (1a).

Eq. (7) is equivalent to (1a). Since (1a) is a regular descriptor system by assumption, $\mathcal{M}_i$ is (a subset of) the configuration space $\mathcal{R}$ and $\tilde{f}$ a vector field corresponding to (1a). Eq. (7) is obtained from (1a) by differentiation and algebraic operations. *No state transformation is involved*!

Note that our reduction algorithm is a generalization of the reduction introduced by Rheinbold [9]. In contrast to [9] we avoid an algebraically overdetermined system and our reduction algorithm results in a system of explicit differential equations.

4. An observation system for a descriptor system

In linear system theory one distinguishes between $\mathcal{R}$-observability, impulse-observability and (complete) observability [1]. Since (1a) is a system without inputs we will restrict our attention to the dynamics in the reachable set. Thus, we are not interested in the impulse behaviour of the system. Following [1, 4] we define:

Definition 3. (1) System (1a) is *observable in the reachable set* (*$\mathcal{R}$-observable*) iff for any $x^1, x^2 \in \mathcal{R}$, $x^1 \neq x^2$, there exists $T>0$ such that $y(\cdot\,; 0, x^1)_{|[0,T]} \not\equiv y(\cdot\,; 0, x^2)_{|[0,T]}$.

(2) System (1a) is *weakly observable in the reachable set* (weakly *$\mathcal{R}$-observable*) iff for any $x \in \mathcal{R}$, there exists $\varepsilon>0$ such that for any $x^1, x^2 \in \mathcal{R}$, $x^1 \neq x^2$ and $\|x - x^i\| < \varepsilon$ $(i=1,2)$ there exists $T>0$ such that $y(\cdot\,; 0, x^1)_{|[0,T]} \not\equiv y(\cdot\,; 0, x^2)_{|[0,T]}$.

We define a state observer for (1a) as:

Definition 4. Let $\tilde{\mathcal{R}} \subset \mathcal{R}$ be a submanifold of $\mathcal{R}$ which is invariant with respect to (1a) and let $x(t; 0, x^0)$ exist for all $x^0 \in \tilde{\mathcal{R}}$ and all $t>0$.

$\hat{x} : \mathbb{R} \times \mathbb{R}^n \times C(\mathbb{R}, \mathbb{R}^m) \to \mathbb{R}^n$ is called a *state observer for* (1a) on $\tilde{\mathcal{R}}$ iff there exists $\varepsilon>0$ such that for any $x^0 \in \tilde{\mathscr{R}}$ and any $\hat{x}^0 \in \mathbb{R}^n$, $\|x^0 - \hat{x}^0\| < \varepsilon$: $\lim_{t\to\infty} \|x(t;0,x^0) - \hat{x}(t;\hat{x}^0, y(\cdot\,;0,x^0))\| = 0$.

According to Definition 4, $\hat{x}$ is not restricted to $\mathscr{R}$. If (1a) is a system with index i and (7) is a differential equation corresponding to (1a), we define an *observation system corresponding to* (1a) as

$$\dot{z} = \tilde{f}(z),$$

$$v = \begin{pmatrix} h(z) \\ y(z) \\ f_{2,1}(z) \\ \vdots \\ f_{2,i}(z) \end{pmatrix} =: \tilde{h}(z), \quad z \in U. \tag{8}$$

z is used to denote the state in (8) in order to distinguish easily between the state of the original descriptor system and the state of the corresponding observation system.

Theorem 1. *Let $A \in C^k(\mathbb{R}^n, \mathbb{R}^{n\times n})$, $f \in C^k(\mathbb{R}^n, \mathbb{R}^n)$, and* (1a) *a system of index i with configuration space $\mathscr{R} \subset \mathbb{R}^n$. The nonlinear descriptor system* (1a) *is weakly observable in $\mathscr{R}$ if the corresponding observation system* (8) *is weakly observable in a neighborhood of $\mathscr{R}$ in $\mathbb{R}^n$.*

Proof. Trivial. □

Using Theorem 1 we propose the following way to observe the state of a nonlinear descriptor system (1a).

First we determine an observation system corresponding to (1a) and check whether the corresponding system (8) is observable. A state z in the configuration space $\mathscr{R}$ would reveal an output $v = (h(z),\ 0)^{\mathrm{T}}$. Since the state x of the descriptor system (1a) naturally is in $\mathscr{R}$, the output y of (1a) and the output v of (8) become compatible if y is augmented with "0" to match the dimension of v. If a state observer $\hat{z}(\cdot\,;\cdot,\cdot)$ for (8) exists, the observer state $\hat{z}$ will asymptotically approach the state x of the descriptor system (1a) if the augmented output $(y(\cdot\,;0,x^0),\ 0)^{\mathrm{T}}$ is used as the third input for $\hat{z}(\cdot\,;\cdot,\cdot)$. If an observer output in $\mathscr{R}$ is required, the observation of (8) may be followed by a projection of $\hat{z}$ onto $\mathscr{R}$, e.g. $\hat{x} = \mathscr{P}(\hat{z}) := \arg\min_{x\in\mathscr{R}} \|x - \hat{z}\|$.

Theorem 2. *Let $A \in C^k(\mathbb{R}^n, \mathbb{R}^{n\times n})$, $f \in C^k(\mathbb{R}^n, \mathbb{R}^n)$, and* (1a) *be a system of index i with configuration space $\mathscr{R} \subset \mathbb{R}^n$ and let $\mathscr{N}$ be a neighborhood of $\mathscr{R}$ in $\mathbb{R}^n$. If $\hat{z} : \mathbb{R} \times \mathbb{R}^n \times C(\mathbb{R}, \mathbb{R}^M) \to \mathbb{R}^n$ is a state observer for* (8) *on $\mathscr{N}$ then $\hat{x} : \mathbb{R} \times \mathbb{R}^n \times C(\mathbb{R}, \mathbb{R}^m) \to \mathbb{R}^n$ defined by*

$$\hat{x}(t;\hat{x}^0, y(\cdot\,;0,x^0)) := \hat{z}\left(t;\hat{x}^0, \begin{pmatrix} y(\cdot\,;0,x^0) \\ 0 \end{pmatrix}\right)$$

is a state observer for (1a) *on $\mathscr{R}$.*

Proof. By Definition 4, $z(t;0,z^0)$ exists for all $z^0 \in \mathscr{N}$ and $t > 0$. Let $x^0 \in \mathscr{R}$ and $\hat{x}^0 \in \mathscr{N}$ with $\|x^0 - \hat{x}^0\| < \varepsilon$, ε according to Definition 3. Then $z(t;0,x^0) = x(t;0,x^0)$ and thus $z(t;0,x^0) \in \mathscr{R}$ for all $t > 0$, as well as $v(t;0,x^0) = (y(t;0,x^0),\ 0)^{\mathrm{T}}$ for all $t > 0$. By assumption, $\lim_{t\to\infty} \|z(t;0,x^0) - \hat{z}(t;\hat{x}^0, v(\cdot\,;0,x^0))\| = 0$ which concludes the proof. □

5. An example

The applicability of the proposed observer design will be demonstrated for a system, which describes a two-link robot manipulator with its end effector fixed to a circular motion. The dynamics of the manipulator are given by

$$M(\theta)\ddot{\theta} = -C(\theta,\dot{\theta}) - k(\theta) + \tau(t) + J^{\mathrm{T}}(\theta)f, \tag{9}$$

with angles $\theta \in \mathbb{R}^2$. The restriction to the circular motion with center $c \in \mathbb{R}^2$ and radius $r \in \mathbb{R}^+$ is given by

$$\tilde{\Gamma}(\eta) := \|\eta - c\|_2^2 - r^2 = \tilde{\Gamma}(H(\theta)) = \Gamma(\theta) = 0, \tag{10}$$

where $H(\theta)$ denotes the transformation from link coordinates to Cartesian coordinates.

Together with the measurement $y = \theta_1$, (9) and (10) yield the system

$$\begin{pmatrix} I_2 & 0 & 0 \\ 0 & M(\theta) & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{\omega} \\ \dot{\lambda} \end{pmatrix} = \begin{pmatrix} \omega \\ -C(\theta,\omega) - k(\omega) + \tau(t) + J^{\mathrm{T}}(\theta)D^{\mathrm{T}}(\theta)\lambda \\ \Gamma(\theta) \end{pmatrix}, \tag{11a}$$

$$y = \theta_1, \tag{11b}$$

with angle velocities $\omega = \dot{\theta} \in \mathbb{R}^2$, and normal contact pressure $\lambda \in \mathbb{R}$ between end effector and circle.

Since (11a) is a system described by Euler–Lagrange equations with a holonomic (geometric) constraint, (11a) naturally reveals (global) index 3 [3, 9]. With $x = (\theta, \omega, \lambda)^{\mathrm{T}}$, index reduction according

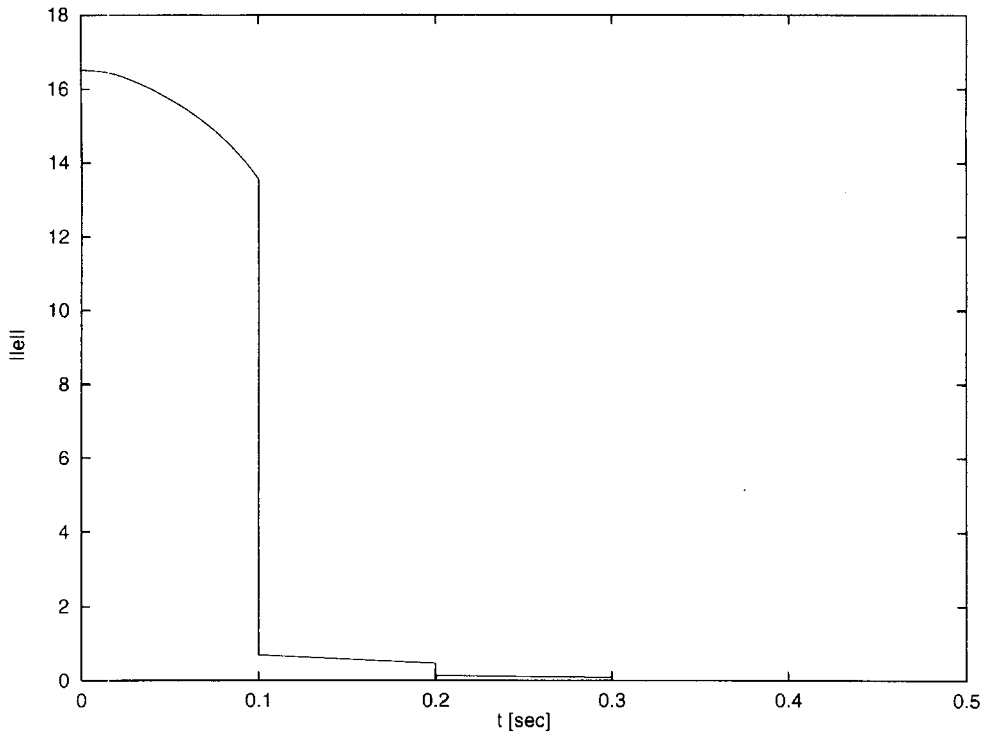

Fig. 1. State observation without projection onto $\mathscr{R}$, $\|e\| = (\Sigma_{i=0}^{5}(x_i - \hat{z}_i)^2)^{1/2}$.

to Section 3 yields the additional constraint equations $0 = \Gamma^{(1)}(x)$ and $0 = \Gamma^{(2)}(x)$. We obtain the configuration space

$$\mathscr{R} = \{x \in \mathbb{R}^5 \mid \Gamma(x) = \Gamma^{(1)}(x) = \Gamma^{(2)}(x) = 0\}$$

and the corresponding observation system

$$\begin{pmatrix} \dot{\theta} \\ \dot{\omega} \\ \dot{\lambda} \end{pmatrix} = \begin{pmatrix} I_2 & 0 & 0 \\ 0 & M(\theta) & 0 \\ \dfrac{\partial \Gamma^{(2)}(x)}{\partial \theta} & \dfrac{\partial \Gamma^{(2)}(x)}{\partial \omega} & \dfrac{\partial \Gamma^{(2)}(x)}{\partial \lambda} \end{pmatrix}^{-1}$$

$$\times \begin{pmatrix} \omega \\ -C(\theta,\omega) - k(\omega) + \tau(t) + J^{\mathrm{T}}(\theta) D^{\mathrm{T}}(\theta)\lambda \\ -\dfrac{\partial \tau(t)}{\partial t} \end{pmatrix},$$

$$y = (\theta_1,\ \Gamma(x),\ \Gamma^{(1)}(x),\ \Gamma^{(2)}(x))^{\mathrm{T}}.$$

For a simulation we choose the initial conditions $\theta_1 = 3\pi/3$ and $\omega_1 = 0$. Considering the constraints implied by $\mathscr{R}$, the resulting initial state is $x^0 = (1.1780, 0.7636, 0, 0, 4.7895)^{\mathrm{T}}$. The implicit torque was set to

$$\tau = M(\theta) \begin{pmatrix} -p_1\theta_1 - 1.178 \\ -p_2\theta_2 \end{pmatrix}.$$

The state observation was carried out with the nonlinear state observer described in [11]. Fig. 1 depicts the results obtained by a state observation without projection onto $\mathscr{R}$.

6. Conclusions

In this paper we proposed a way to observe the state of a nonlinear descriptor system given in quasilinear form. It was shown that the problem of observing the state of the descriptor system can be transformed into the problem of observing the state of a nonlinear system with an explicit differential equation. The transformation of the problem is carried out by differentiations and algebraic manipulations. No nonlinear state transformation is required.

Once the problem is rewritten, the actual state observation can be carried out by using an arbitrary nonlinear state observer for an explicit nonlinear system.

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