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April 23, 2018

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Outline

3.36pt

Repetition

Supervisory Control Algorithms

Heuristic Control Approaches

Optimal Control Strategies

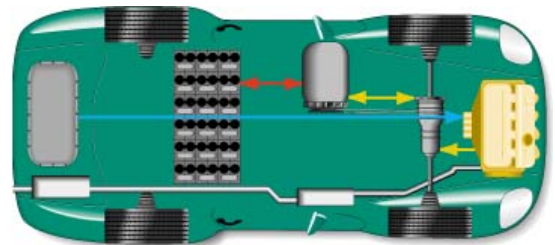
Analytical solutions to Optimal Control Problems  
ECMS – Equivalent Consumption Minimization Strategy

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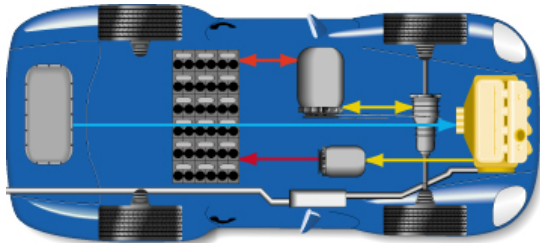
Hybrid Electrical Vehicles – Parallel

- ▶ Two parallel energy paths
- ▶ One state in QSS framework, state of charge



Hybrid Electrical Vehicles – Serial

- ▶ Two paths working in parallel
- ▶ Decoupled through the battery
- ▶ Two states in QSS framework, state of charge & Engine speed

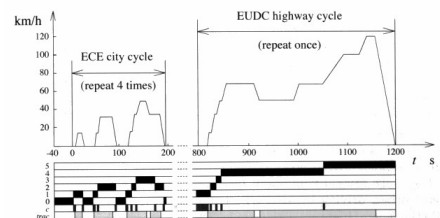


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Optimization, Optimal Control, Dynamic Programming

What gear ratios give the lowest fuel consumption for a given driving cycle?

–Problem presented in appendix 8.1



Problem characteristics

- ▶ Countable number of free variables,  $i_{g,j}, j \in [1, 5]$
- ▶ A “computable” cost,  $m_f(\dots)$
- ▶ A “computable” set of constraints, model and cycle
- ▶ The formulated problem

$$\begin{aligned} \min_{i_{g,j}, j \in [1,5]} & m_f(i_{g,1}, i_{g,2}, i_{g,3}, i_{g,4}, i_{g,5}) \\ \text{s.t.} & \text{ model and cycle is fulfilled} \end{aligned}$$

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Optimal Control – Problem Motivation

Car with gas pedal  $u(t)$  as control input:  
How to drive from A to B on a given time with minimum fuel consumption?

- ▶ Infinite dimensional decision variable  $u(t)$ .
- ▶ Cost function  $\int_0^{t_f} \dot{m}_f(t) dt$
- ▶ Constraints:
  - ▶ Model of the car (the vehicle motion equation)

$$\begin{aligned} m_v \frac{d}{dt} v(t) &= F_t(v(t), u(t)) - (F_a(v(t)) + F_r(v(t)) + F_g(x(t))) \\ \frac{d}{dt} x(t) &= v(t) \\ \dot{m}_f &= f(v(t), u(t)) \end{aligned}$$

- ▶ Starting point  $x(0) = A$
- ▶ End point  $x(t_f) = B$
- ▶ Speed limits  $v(t) \leq g(x(t))$
- ▶ Limited control action  $0 \leq u(t) \leq 1$

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General problem formulation

- ▶ Performance index

$$J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t) dt$$

- ▶ System model (constraints)

$$\frac{d}{dt} x = f(x(t), u(t), t), x(t_a) = x_a$$

- ▶ State and control constraints

$$\begin{aligned} u(t) &\in U(t) \\ x(t) &\in X(t) \end{aligned}$$

## Dynamic programming – Problem Formulation

- ▶ Optimal control problem

$$\min J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t) dt$$

$$\text{s.t. } \frac{d}{dt} x = f(x(t), u(t), t)$$

$$x(t_a) = x_a$$

$$u(t) \in U(t)$$

$$x(t) \in X(t)$$

- ▶  $x(t), u(t)$  functions on  $t \in [t_a, t_b]$
- ▶ Search an approximation to the solution by discretizing
  - ▶ the state space  $x(t)$
  - ▶ and maybe the control signal  $u(t)$
 in both amplitude and time.
- ▶ The result is a combinatorial (network) problem

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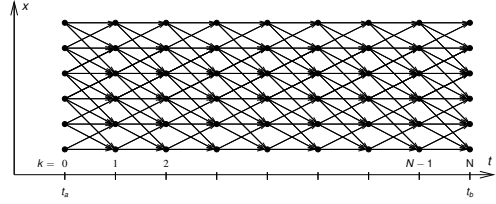
## Deterministic Dynamic Programming – Basic algorithm

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$

$$x_{k+1} = f_k(x_k, u_k)$$

Algorithm idea:

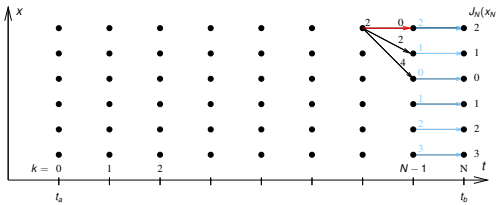
Start at the end and proceed backwards in time to evaluate the optimal cost-to-go and the corresponding control signal.



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## Deterministic Dynamic Programming – Basic Algorithm

Graphical illustration of the solution procedure



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## Arc Cost Calculations

There are two ways for calculating the arc costs

- ▶ Calculate the exact control signal and cost for each arc.
  - Quasi-static approach
- ▶ Make a grid over the control signal and interpolate the cost for each arc.
  - Forward calculation approach

Matlab implementation – it is important to utilize matrix calculations

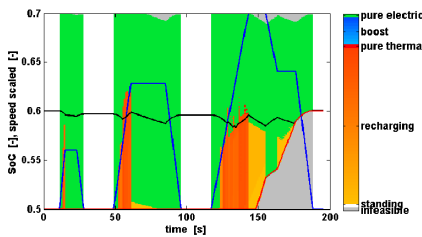
- ▶ Calculate the whole bundle of arcs in one step
- ▶ Add boundary and constraint checks

2D and 3D grid examples on whiteboard

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## Parallel Hybrid Example

- ▶ Fuel-optimal torque split factor  $u(SOC, t) = \frac{T_{e-motor}}{T_{gearbox}}$
- ▶ ECE cycle
- ▶ Constraints  $SOC(t = t_f) \geq 0.6, SOC \in [0.5, 0.7]$



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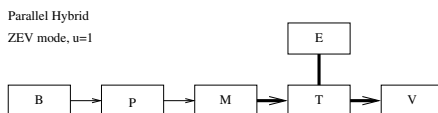
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## Parallel Hybrid – Modes and Power Flows

The different modes for a parallel hybrid

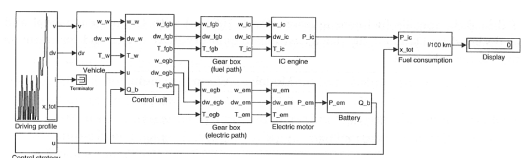
$$u \approx P_{batt} / P_{vehicle}$$

Battery drive mode (ZEV)



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## Control algorithms



- ▶ Determining the power split ratio  $u$

$$u_j(t) = \frac{P_j(t)}{P_{m+1}(t) + P_l(t)} \quad (4.110)$$

- ▶ Clutch engagement disengagement  $B_c \in \{0, 1\}$
- ▶ Engine engagement disengagement  $B_e \in \{0, 1\}$

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Power split  $u$ , Clutch  $B_c$ , Engine  $B_e$

	Mode	$u$	$B_e$	$B_c$
1	ICE	0	1	1
2a	ZEV	1	0	0
2b	ZEV	1	0	1
3	Power assist	$[0,1]$	1	1
4	Recharge	$< 0$	1	1
5a	Regenerative braking	1	0	0
5a	Regenerative braking	1	0	1

All practical control strategies have engine shut off when the torque at the wheels are negative or zero; standstill, coasting and braking.

- ▶ Non-causal controllers
  - ▶ Detailed knowledge about future driving conditions.
  - ▶ Position, speed, altitude, traffic situation.
  - ▶ Uses: Regulatory drive cycles, public transportation, long haul operation, GPS based route planning.
- ▶ Causal controllers
  - ▶ No knowledge about the future...
  - ▶ Use information about the current state.
  - ▶ Uses: "The normal controller", on-line, in vehicles without planning

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Classification II – Vehicle Controllers

Some Comments About the Problem

- ▶ Heuristic controllers
  - Causal
  - State of the art in most prototypes and mass-production
- ▶ Optimal controllers
  - Often non-causal
  - Solutions exist for simplifications
- ▶ Sub-optimal controllers
  - Often causal

- ▶ Difficult problem
- ▶ Unsolved problem for causal controllers
- ▶ Rich body of engineering reports and research papers on the subject
  - This can clearly be seen when reading chapter 7!

On-going work to include optimal controllers in prototypes

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Heuristic Control Approaches

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Operation usually depends on a few vehicle operation

- ▶ Rule based:
  - Nested if-then-else clauses
  - if  $v < v_{low}$  then use electric motor ( $u=1$ ).
  - else...
- ▶ Fuzzy logic based
  - Classification of the operating condition into fuzzy sets.
  - Rules for control output in each mode.
  - Defuzzification gives the control output.

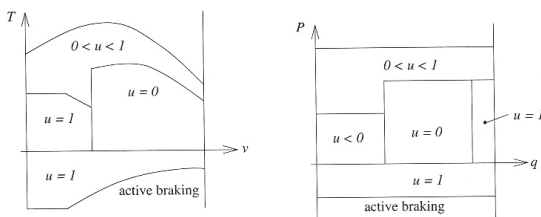
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Heuristic Control Approaches

Heuristic Control Approaches – Concluding Remarks

- ▶ Parallel hybrid vehicle (electric assist)



- ▶ Determine control output as function of some selected state variables:
  - vehicle speed, engine speed, state of charge, power demand, motor speed, temperature, vehicle acceleration, torque demand.

- ▶ Easy to conceive
- ▶ Relatively easy to implement
- ▶ Result depends on the thresholds
- ▶ Proper tuning can give good fuel consumption reduction and charge sustainability
- ▶ Performance varies with cycle and driving condition
  - Not robust
- ▶ Time consuming to develop an tune for advanced hybrid configurations

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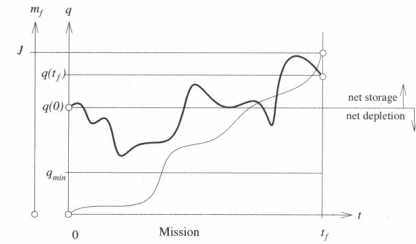
ECMS – Equivalent Consumption Minimization Strategy

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## Consider a driving mission

### ► Variables.

Control signal –  $u(t)$ , System state –  $x(t)$ , State of charge –  $q(t)$  (is a state).



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## Formulating the Optimal Control Problem

–What is the optimal behaviour? Defines *Performance index J*.

- Minimize the fuel consumption

$$J = \int_0^{t_f} \dot{m}_f(t, u(t)) dt$$

- Balance between fuel consumption and emissions

$$J = \int_0^{t_f} \left[ \dot{m}_f(t, u(t)) + \alpha_{CO} \dot{m}_{CO}(x(t), u(t)) + \alpha_{NO} \dot{m}_{NO}(x(t), u(t)) + \alpha_{HC} \dot{m}_{HC}(x(t), u(t)) \right] dt$$

- Include driveability criterion

$$J = \int_0^{t_f} \dot{m}_f(t, u(t)) + \beta \left( \frac{d}{dt} a(t) \right)^2 dt$$

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## First Solution to the Problem

- Minimize the fuel consumption

$$J = \int_0^{t_f} \dot{m}_f(t, u(t)) dt$$

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## Including constraints

- Hard or soft constraints

$$\min J(u) = \int_0^{t_f} L(t, u(t)) dt$$

s.t.  $q(0) = q(t_f)$

$$\min J(u) = \phi(q(t_f)) + \int_0^{t_f} L(t, u(t)) dt$$

- How to select  $\phi(q(t_f))$ ?

$$\phi(q(t_f)) = \alpha (q(t_f) - q(0))^2$$

penalizes high deviations more than small, independent of sign

$$\phi(q(t_f)) = w (q(0) - q(t_f))$$

penalizes battery usage, favoring energy storage for future use

- One more feature from the last one

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## Including constraints

- Including battery penalty according to

$$\phi(q(t_f)) = w (q(0) - q(t_f)) = -w \int_0^{t_f} \dot{q}(t) dt$$

enables us to rewrite

$$\min J(u) = \int_0^{t_f} L(t, u(t)) - w \dot{q}(t) dt$$

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## Constraints That are Also Included

- State equation  $\dot{x} = f(x)$  is also included – From Lecture 5
- Consider hybrid with only one state, SoC

$$\min J(u) = \phi(q(t_f), t_f) + \int_0^{t_f} L(t, u(t)) dt$$

$$\text{s.t. } \frac{d}{dt} q = f(t, q(t), u(t))$$

$$u(t) \in U(t)$$

$$q(t) \in Q(t)$$

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- Core of the problem

$$\min J(u) = \phi(q(t_f), t_f) + \int_0^{t_f} L(t, u(t)) dt$$

$$s.t. \dot{q}(t) = f(t, q(t), u(t))$$

- Hamiltonian from optimal control theory

$$H(t, q(t), u(t), \mu(t)) = L(t, u(t)) + \mu(t) f(t, q(t), u(t))$$

- Hamiltonian

$$H(t, q(t), u(t), \mu(t)) = L(t, u(t)) + \mu(t) f(t, q(t), u(t))$$

- Solution (theory from Appendix B)

$$u(t) = \arg \min_u H(t, q(t), u(t), \mu(t))$$

with

$$\dot{\mu}(t) = - \frac{\partial}{\partial q} H(t, q(t), u(t))$$

$$\dot{q}(t) = f(t, q(t), u(t))$$

- If  $\frac{\partial}{\partial q} H(t, q(t), u(t)) = 0$  the problem becomes simpler  $\mu$  becomes a constant  $\mu_0$ , search for it when solving

Analytical Solutions to Optimal Control Problems

- $\mu_0$  depends on the (soft) constraint

$$\mu_0 = \frac{\partial}{\partial q(t_f)} \phi(q(t_f)) = \text{/special case/} = -w$$

- Different efficiencies

$$\mu_0 = \frac{\partial}{\partial q(t_f)} \phi(q(t_f)) = \begin{cases} -w_{dis}, & q(t_f) > q(0) \\ -w_{chg}, & q(t_f) < q(0) \end{cases}$$

- Introduce equivalence factor (scaling) by studying battery and fuel power

$$s(t) = -\mu(t) \frac{H_{LHV}}{V_b Q_{max}}$$

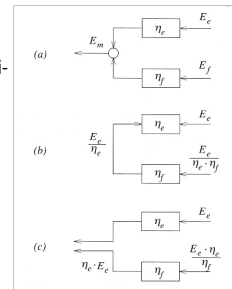
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Determining Equivalence Factors I

Constant engine and battery efficiencies

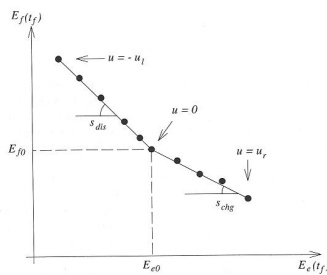
$$s_{dis} = \frac{1}{\eta_e \eta_f}$$

$$s_{chg} = \frac{\eta_e}{\eta_f}$$



Determining Equivalence Factors II

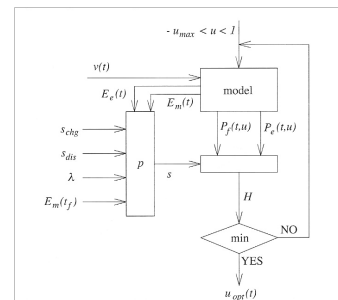
- Collecting battery and fuel energy data from test runs with constant  $u$  gives a graph



- Slopes determine  $s_{dis}$  and  $s_{chg}$ .

ECMS On-line Implementation

Flowchart



There is also a T-ECMS (telemetry-ECMS)