Observer Design and Model Augmentation for Bias Compensation Applied to an Engine

Erik Höckerdal [*,**], **Erik Frisk** [*], **and Lars Eriksson** [*]

[*] *Department of Electrical Engineering, Linköpings universitet, Sweden, {hockerdal,frisk,larer}@isy.liu.se*
[**] *Scania CV AB, Södertälje, Sweden, erik.hockerdal@scania.com*

Abstract: A systematic design method for reducing bias in observers is developed. The method utilizes an observable default model of the system together with measurement data from the real system and estimates a model augmentation. The augmented model is then used to design an observer which reduces the estimation bias compared to a default observer. A key result is the theoretical analysis that characterizes the possible augmentations is also conducted. The method is applied to a truck engine where the resulting augmented observer reduces the estimation bias with 50 % in an ETC.

Keywords: bias compensation, EKF, non-linear, observer

1. INTRODUCTION

In all model based control or diagnosis systems, the performance of the system is directly dependent on the accuracy of the model. Further, modeling is time consuming and, even if much time is spent on physical modeling, there will always be errors in the model. This is especially true if there are constraints on the model complexity, as is the case in most real time systems. Another scenario is that a model developed for some purpose, e.g. control, exists but needs corrections before it can be used in for example diagnosis.

In many applications, for example engine control and engine diagnosis, it is crucial to have unbiased estimates. In model based diagnosis, the true system is often monitored by comparing measured signals to estimated signals. If the magnitude of the difference, the residual, is above a certain limit a decision that something is wrong is made. In engine control, the goal is to maximize torque output while keeping the emissions below legislated levels and the fuel consumption as low as possible. For diesel engines this is especially hard since the control system does not have any feedback information from a λ- or NO_x-sensor and have to rely on estimated signals instead. In both cases, biased estimates impairs the performance.

The objective of this work is to develop a systematic method for reducing estimation bias in model based observers without involving further modeling efforts.

The model utilizes an observable model of, and measurement data from a true system. The given model, referred to as the default model, and the measured inputs and outputs from the true system are used to estimate a suitable model augmentation. Then, the augmented model is used to design an observer that is shown to give estimates with reduced bias compared to an observer based on the default model. A key result is a theoretical characterization of all possible augmentations. Finally the method is evaluated on a non-linear diesel engine model with experimental data from an engine test cell.

2. PROBLEM FORMULATION

Previous experience at Scania CV AB of state estimation based on an existing state-space model of a truck engine reveals that the model captures dynamic behavior reasonably well but suffers from stationary errors. Designing an observer based on this model results in biased estimates and how to reduce this problem in a systematic manner is the topic of this paper.

The starting point is an existing model, referred to as the default model, that is provided in state-space form

$$\dot{x} = f(x, u) \tag{1a}$$
$$y = h(x), \tag{1b}$$

where x is the state-vector, u the known control inputs, y the measurement vector, and f and h are non-linear functions.

The objective is to find a systematic way to design an observer that gives an unbiased estimate of either the complete state x or a function of the state $z = g(x)$. This should be done even though the default model is subjected to significant bias errors. A direct approach to compensate for constant, or slowly varying, biases is to augment the default model with bias variables q as

$$\dot{x} = \tilde{f}(x, u, q) \tag{2a}$$
$$\dot{q} = 0 \tag{2b}$$
$$y = \tilde{h}(x, q) \tag{2c}$$

and design the observer using this augmented model. If the augmentation captures the true modeling errors and the augmented system is observable, the observer estimates can be made unbiased.

An obvious question is then how to introduce the bias variable q in the model equations. One way is through process knowledge but in this paper we propose an estimation procedure based on available measurement data. Besides the natural restriction, that the augmented model (2) is observable, it is also desirable to not introduce more extra bias states than necessary. It is therefore desirable to find a bias vector q with as low dimension as possible that manages to reduce the bias. Another reason for finding a low dimensional bias is that, since the model is a first-principles physical model, bias in multiple states may be explained by one underlying bias affecting all these states. For example, bias in two pressures can originate from a bias in the mass flow between the two volumes or an incorrect modeling of energy conservation can give rise to bias in several states connected to the energy.

In the model (1) there are two natural ways to introduce biases, in the dynamic equation (1a) or in the measurement equation (1b). In the truck engine application the sensors, intake and exhaust manifold pressures and turbine speed, are considered more reliable than the model and the bias augmentation is therefore introduced in the dynamic equations according to

$$\dot{x} = f(x - A_q q, u) \tag{3a}$$
$$\dot{q} = 0 \tag{3b}$$
$$y = h(x). \tag{3c}$$

where a stationary point of the system is moved by $A_q q$. The matrix A_q is thus a description on how the underlying bias variable q influences the stationary value of the state variable x. The model (3) will be referred to as the augmented model.

2.1 Problem outline

Based on the discussion above, the problem studied in the sections to follow can now be stated as: Given a default model (1) and available measurement data, find a low order bias augmented model (3) and design an observer that estimates x with reduced bias compared to using the default model. The observer should also be implementable in an Engine Control Unit (ECU).

To solve the problems, some issues need to be addressed. First, which matrices A_q are at all possible? Not all are possible since we require that the augmented system should be observable and a characterization of possible augmentations is derived in Section 3. Among these possible bias augmentations, which should be used? Section 4 describes three approaches for how to estimate a, for bias compensation, suitable low order A_q based on measurement data. Section 5 finally summarizes the procedure and Section 6 presents two examples of the proposed estimator design methodology applied to a Scania diesel engine using simulated and real measurement data respectively.

2.2 Discretization

As a first step, the nonlinear augmented model (3) is transformed to a linearized time discrete model. A reason for the discretization is the demand on the implementation, which will be done in the ECU as a time discrete system. An Euler forward discretization with step size T_s seconds is used. The reasons for the linearization are, to simplify the observability analysis and to get a model that fits into the EKF frame-work used in the observer design. It is further assumed that conclusions on observability made locally can be used to draw conclusions of the global observability properties of the model. This gives the following model

$$\begin{bmatrix} x_{t+1} \\ q_{t+1} \end{bmatrix} = \begin{bmatrix} I + T_s A & -T_s A A_q \\ 0 & I \end{bmatrix} \begin{bmatrix} x_t \\ q_t \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_t \tag{4a}$$
$$y_t = [C \;\; 0] \begin{bmatrix} x_t \\ q_t \end{bmatrix}, \tag{4b}$$

where

$$A = \left.\frac{\partial f}{\partial x}\right|_{\substack{x=x_0\\u=u_0}}, \quad B = \left.\frac{\partial f}{\partial u}\right|_{\substack{x=x_0\\u=u_0}}, \quad \text{and } C = \left.\frac{\partial h}{\partial x}\right|_{\substack{x=x_0\\u=u_0}}$$

3. POSSIBLE AUGMENTATIONS

Augmenting a model with more states may affect the observability of the model. Since the purpose of the augmented model is to use it for estimation, observability has to be maintained also after the augmentation. To find which augmentations that are possible an observability investigation of the augmented model is performed. The aim is to derive a necessary and sufficient condition on A_q such that the augmented model is observable. The observability criterion used is known as the Popov-Belevitch-Hautus(PBH)-test (Kailath, 1980).

Theorem 1. A pair (C, F) is observable if and only if the matrix

$$\mathcal{O} = \begin{pmatrix} C \\ F - \lambda I \end{pmatrix}$$

has full column rank for all $\lambda \in \mathbb{C}$.

To proceed, two assumptions regarding the default model are made. First, the default model is used for observer design and is therefore assumed to be observable. Second, A is assumed to be invertible, which is the case in the application example. Now, using Theorem 1 and the two assumptions above, the main result of this section can be formulated as

Theorem 2. Assume that (C, A) in (4) is an observable pair and that A is non-singular, then the augmented system is observable if and only if

$$\operatorname{Im} A_q \cap \operatorname{Ker} C = \{0\}$$

which is equivalent to CA_q having full column rank.

Proof. The PBH-test applied to the augmented model (4) gives

$$\mathcal{O}_{\text{aug}} = \left(\begin{array}{c|c} C & 0 \\ I + T_s A - \lambda I & -T_s A A_q \\ \hline 0 & I - \lambda I \end{array}\right)$$

Since the default model is assumed to be observable, the upper left block in $\mathcal{O}_{\text{aug}}$ has full column rank for all λ and $\mathcal{O}_{\text{aug}}$ can lose rank only for $\lambda = 1$. It is therefore sufficient to check the column rank of

$$\begin{pmatrix} C & 0 \\ T_s A & -T_s A A_q \end{pmatrix}$$

Which is equivalent to requiring that the only solution to

$$Cx = 0$$
$$T_s A(x - A_q q) = 0$$

is $x = 0, \; q = 0$. Since A is non-singular this is equivalent to,

$$Cx = 0$$
$$x = A_q q$$

Hence the augmented system is observable if and only if

$$\operatorname{Im} A_q \cap \operatorname{Ker} C = \{0\}$$

or, equivalently, that the matrix CA_q has full column rank. □

This means that the space spanned by the columns in A_q can not lie in the null space of C for the augmented model to be observable.

A closer look at the requirement that CA_q has to have full column rank convey some interesting results. Firstly, it is easily seen that the number of augmented states, $n_q = \dim q$, never can exceed the number of measurement signals, $n_y = \dim y$, i.e. $n_q \leq n_y$. Secondly, imagine a C that has one or several zero columns, then the product CA_q will not contain any information from those rows in A_q corresponding to the zero columns in C. That is, those rows in A_q that correspond to zero columns in C will not contribute to the observability.

Also note that the following results regarding observability are not dependent on the method chosen for discretization. As long as T_s is chosen small enough the results are valid also for, e.g. zero-order-hold (Kalman et al., 1963).

Example 1. Possible augmentations of a small system with invertible A, and

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(1) An augmentation

$$A_q = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \text{ is not observable since } CA_q = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

and does not have full column rank. The reason for this is that the second column in A_q only has non-zero components in the row corresponding to the zero column in C.

(2) However, if either of the two zeros in the second column of A_q is interchanged to, for example a one, the augmentation becomes observable.

$$A_q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow CA_q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

◇

4. AUGMENTATION ESTIMATION

The next question is how to find a suitable augmentation, that fulfills the requirements derived in Section 3, using data (y, u) from the real system. Three approaches for how to estimate a suitable augmentation have been developed. In the following, $I + T_s A$ is substituted for F to increase readability.

4.1 Approach 1

The first approach utilizes the discretized linearization directly,

$$x_{t+1} = F_t x_t + (I - F_t) A_q q_t + B_t u_t$$
$$y_t = C_t x_t$$

Inverting the measurement equation and inserting the resulting x in the dynamic equation, gives

$$A_q q_t = (I - F_t)^{-1} (C_{t+1}^\dagger y_{t+1} - F_t C_t^\dagger y_t - B_t u_t),$$

where † denotes the pseudo inverse. To find a suitable augmentation, the $A_q q_t$'s are collected in a matrix, $R_{A_q q_t} = [A_q q_1, \ \ldots, \ A_q q_N]$, which is analyzed by singular value decomposition (SVD). Here it is crucial that the SNR is high enough, otherwise the noise is a dominating part of $A_q q_t$ and an SVD would give a basis for the noise, not the bias. However, if the SNR is high enough the SVD gives a basis for the space in which the bias moves and $\hat{A}_q$ can be chosen to span that space.

An advantage with this approach over the other two is that there is no need for computing any intermediate observer for estimating the augmentation. A disadvantage, besides that C has to have full column rank, is that, since no filter is involved, it is sensitive to measurement noise.

4.2 Approach 2

The second approach is based on an SVD of the residuals originating from an observer based on the default model. Here, the observer is an extended Kalman filter (EKF) (Kailath et al., 2000), where the noise covariance matrices Q and R are design parameters tuned by the user. The estimation error becomes,

$$\begin{aligned} e_{t+1} &= x_{t+1} - \hat{x}_{t+1|t+1} \\ &= F_t x_t + (I - F_t) A_q q + B_t u_t - \\ &(F_t \hat{x}_{t|t} + B_t u_t + K_t (y_{t+1} - C_t F_t \hat{x}_{t|t} - C_t B_t u_t)) \\ &= \{ y_{t+1} = C_t F_t x_t + C_t (I - F_t) A_q q + C_t B_t u_t \} \\ &= (F_t - K_t C_t F_t) e_t + (I - K_t C_t)(I - F_t) A_q q \end{aligned} \quad (7)$$

Equation (7) requires that the estimation error is known which normally is not the case, hence the residuals,

$$r_t = y_t - \hat{y}_{t|t} = C_t (x_t - \hat{x}_{t|t}) = C_t e_t, \quad (8)$$

are used for estimating an augmentation. The fact that residuals from an observer is used instead of the measurements makes this approach less sensitive to low SNR, compared to Approach 1.

Here, solely stationary parts of the residuals are involved when searching an appropriate augmentation, A_q. In the example the stationary parts are separated out through visual inspection of the data at hand. It would be possible to use also dynamical parts of the residuals and a dynamical inverse. The reason for not utilizing these is to prevent dynamical estimation errors from affecting the estimation of the constant or slowly varying bias. This results in

$$\begin{aligned} r_{\text{stat}} &= C_{\text{stat}} e_{\text{stat}} \\ &= C_{\text{stat}} (I - F_{\text{stat}} + K_{\text{stat}} C_{\text{stat}} F_{\text{stat}})^{-1} \times \\ &(I - K_{\text{stat}} C_{\text{stat}})(I - F_{\text{stat}}) A_q q_{\text{stat}} \end{aligned}$$

According to this A_q can be found by first finding the stationary residuals in a set of system operating points, collect these in the same way as in Approach 4.1, and perform an SVD. The SVD returns a basis for the residuals, V_r, and A_q can be estimated as

$$\begin{aligned} \hat{A}_q = (C_{\text{stat}} (I - F_{\text{stat}} + K_{\text{stat}} C_{\text{stat}} F_{\text{stat}})^{-1} \times \\ (I - K_{\text{stat}} C_{\text{stat}})(I - F_{\text{stat}}))^\dagger V_r \end{aligned} \quad (9)$$

4.3 Approach 3

An alternative to Approach 2 for finding A_q is to augment the default model with as many extra states as possible. According to Theorem 2, CA_q has to have full column rank. This means that A_q can have a maximum of n_y columns, one non-zero element per column, and these non-zero elements have to correspond to non-zero columns of C. Run the observer based on the augmented model, perform an SVD on the stationary parts of the augmented states, and assemble A_q.

An advantage with this approach is that no inversions as those in (9) are needed. A disadvantage is that the order of the observer may become quite large during the augmentation estimation. In the worst case the order of the augmented model will be twice the order of the default model.

Example 2. Here the maximum possible augmentation is illustrated for a default model with

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Let $\times$ denote a non-zero element, then some possible augmentations are

$$A_q^1 = \begin{bmatrix} \times & 0 \\ 0 & \times \\ 0 & 0 \end{bmatrix}, \text{ and } A_q^2 = \begin{bmatrix} 0 & \times \\ \times & 0 \\ 0 & 0 \end{bmatrix}$$

since

$$CA_q^1 = \begin{bmatrix} \times & 0 \\ 0 & \times \end{bmatrix}, \text{ and } CA_q^2 = \begin{bmatrix} 0 & \times \\ \times & 0 \end{bmatrix},$$

which have full column rank. While an augmentation

$$A_q^3 = \begin{bmatrix} \times & 0 \\ 0 & 0 \\ 0 & \times \end{bmatrix} \text{ is not possible since } CA_q^3 = \begin{bmatrix} \times & 0 \\ 0 & 0 \end{bmatrix}$$

does not have full column rank. ◇

4.4 Remarks

The SVD returns a matrix, V_r, containing orthogonal vectors spanning the space in which the bias moves and the corresponding singular values. The singular values constitute the diagonal of a matrix, S_r and the i:th diagonal element corresponds to the i:th column in V_r. The singular values in S_r are ordered in descending order which means that the far left columns, corresponding to large singular values, represent the most dominating directions along in the space in which the bias moves. Therefore the dimension of q can be found by comparing the singular values in S_r, and picking the most significant ones. Then the corresponding columns of V_r are used in the estimation of $\hat{A}_q$.

Also note that, according to the discussion in the end of Section 3, the properties of C place restrictions on which A_q:s that are possible to find. The conclusion of that discussion is that rows in A_q corresponding to zero columns in C become zero in the estimation step. As a consequence, the observer based on an estimated augmentation may not be able to reduce the bias in the estimates to acceptable levels. This problem can be circumvented in, for example one of the two following ways. The first is for an engineer to design an A_q not possible to find through estimation, for example through knowledge of the underlying physics. The second is to add extra sensors to the true system to acquire a full column rank C which enables estimation of all rows in A_q.

The example below illustrates the remarks regarding the affects the properties of C have on the augmentation estimation.

Example 3. Consider a true system with

$$F = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

and a true bias,

$$A_q = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Then the estimation of A_q, according to (9), will have the following structure

$$\hat{A}_q = \begin{bmatrix} \times \\ \times \\ 0 \end{bmatrix}$$

That is, rows in $\hat{A}_q$ corresponding to zero columns in C can not be estimated. ◇

5. METHOD

The procedure can be summarized in three steps.

Step 1 - Linearize and discretize the model if necessary. Normally, the default model is a non-linear time continuous model, (1), and has to be linearized and discretized. There are several ways to discretize a model, Euler forward/backward, central difference etc, that have different stability properties.

Step 2 - Find an appropriate augmentation, A_q, and compile an augmented model (4). Here the designer has a choice, either to estimate an augmentation from measured data using one of the three approaches presented in Section 4, or introduce an augmentation found in some other way. With good knowledge of the system, the designer might have some idea of what is causing the bias in the estimates and can chose an appropriate A_q.

Step 3 - Design an observer based on the augmented model (3) and the A_q found in Step 2.

6. EXPERIMENTAL EVALUATION

To evaluate the approach two experiments are performed on a non-linear model of a truck engine.

In the first experiment the method is applied to synthetic data created by introducing known biases in a non-linear model of a Diesel engine with three states. The states, x_1, x_2, and x_3, represent intake and exhaust manifold pressures, and turbine speed respectively, see Appendix A. In the second experiment, real data from the engine is used together with the engine model to illustrate the gain in a real application.

In many real applications it is convenient to have a filter when estimating an augmentation to reduce the influence of measurement noise, therefore approach 2 is chosen in both these experiments. The observer based on the default model is referred to as the default observer while the observer based on the augmented model is referred to as the augmented observer.

6.1 Simulation study

The introduced bias is represented by

$$A_q = \begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 0 & 0 \end{bmatrix}$$

and two slowly varying biases q_1 and q_2. This A_q means that there are two independent biases affecting the pressures in the model which varies between approximately 0 and 10 % of the state values. The default system has the linear measurement equation where $y_1 = x_1$ and $y_2 = x_3$. However, according to the discussion in Section 4.4, an augmentation as the one introduced in this example can not be estimated without a direct connection between x_2 and y. Therefore the measurement equation is extended with an extra sensor for x_2. To make the simulation more realistic, white system and measurement noise are added in the creation of the synthetic data. Using the synthetic data and the default model the augmentation estimation results in

$$S_r \approx \begin{bmatrix} 136 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}, \text{ and } \hat{A}_q \approx \begin{bmatrix} 0.892 & -0.452 \\ 0.452 & 0.892 \\ 0.017 & 0.006 \end{bmatrix}$$

where S_r indicates that there are two slowly varying biases present. Hence, $\hat{A}_q$ is estimated using the first two columns of V_r and (9).

At a first look $\hat{A}_q$ does not appear similar to A_q. However, the crucial fact is that the columns of $\hat{A}_q$ and A_q span, approximately, the same space. A closer look reveals that the elements in the bottom row is significantly smaller than the other elements, and that the factor between row one and two are approximately 2. That is, the only thing that differs, besides a scaling, is that the signs do not match.

An observer is created using EKF methodology and a model augmented according to this estimated $\hat{A}_q$. The performance is compared to the default observer. The state estimates of x_1 and x_2 are presented in Figure 1 together with the true states. The reason for not presenting x_3 is lack of space and that the quality of the estimates are comparable to the estimates of x_1.

It can be seen that the augmented observer estimates x_2 better than the default observer while they both seem to estimate x_1 equally well.

Since it is hard to make any further observations regarding the estimate of x_1 based on the estimates themselves the estimation errors are plotted in Figure 2. Here it can be seen that all estimates become better with the augmented observer than with the default observer.

6.2 Application to real measurement data

In a final experiment the procedure is applied to a real application where the exhaust manifold pressure is estimated without using a sensor measuring it. The exhaust manifold pressure is chosen since it is hard to measure due

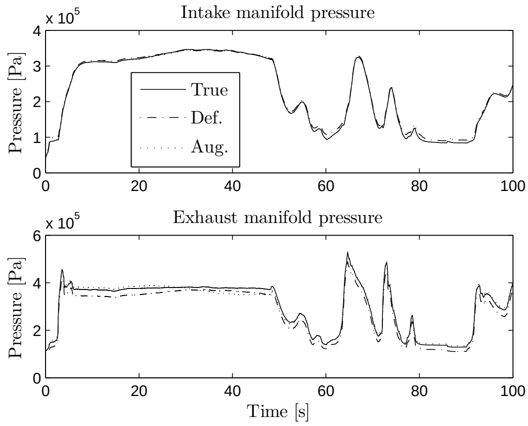

Fig. 1. True states and estimated states using default and augmented observer in the simulation study.

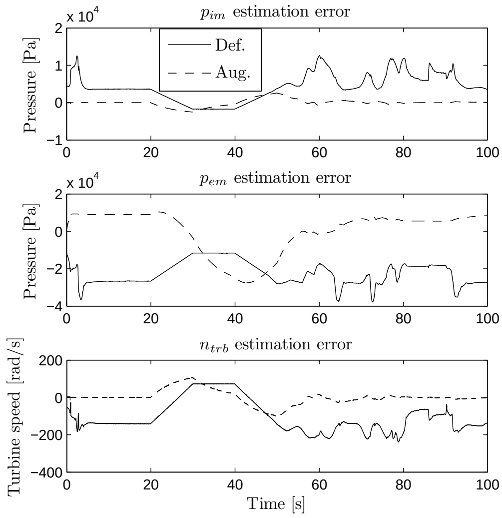

Fig. 2. Estimation errors using default and augmented observer in the simulation study.

to strong pulsations in the exhaust gases. An augmentation is estimated using data from two stationary operating points in the European transient cycle (ETC) of about 1500 samples each resulting in

$$S_r \approx \begin{bmatrix} 1303 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 0.92 \end{bmatrix}, \text{ and } \hat{A}_q \approx \begin{bmatrix} -0.981 \\ 0.186 \\ -0.051 \end{bmatrix}$$

where S_r indicates that there is one dominant slowly varying bias present. Hence, $\hat{A}_q$ is estimated using the first column of V_r and (9).

In this example an exhaust pressure sensor is used both for the estimation of $\hat{A}_q$ and in the evaluation of the augmented observer. However, note that the extra pressure sensor is not used in the augmented observer. This illustrates that additional sensors can be utilized in the design steps to gain more knowledge about the system. Here the additional sensor provides valuable information when $\hat{A}_q$ is estimated since there is no prior knowledge of what states the bias influences. This is natural since most information can be extracted if all states are measured during the design, however it is not a necessity for the

Table 1. Mean and maximum estimation errors using default and augmented observer for the application to real measurement data.

	Max abs. error		Mean error	
	Def.	Aug.	Def.	Aug.
x_1 [Pa]	5430	5220	-931	-794
x_2 [Pa]	280534	289219	-20112	-11163
x_3 [rad/s]	1217	1220	18.88	-10.52

proposed procedure. Utilizing this possibility one must be cautious and check the observability of the augmented system that does not rely on the additional sensors that are used for estimating $\hat{A}_q$.

The augmented observer is compared to the default observer. Here the true states are approximated with non-causal, low-pass filtered measurements, where the filter has a cut off frequency of 2 Hz. In Table 1 it can be seen that the mean errors are about 50 % smaller for the augmented observer than for the default observer while the maximum absolute errors are approximately the same. Note that an ETC is a quite dynamic cycle and does not contain many stationary parts. With this in mind, and the fact that the aim of the method is to reduce stationary bias, a reduction of the mean error with about 50 % in an ETC is a promising result. Figure 3 shows the state estimates and Figure 4 shows the estimation errors. In Figure 4 it can be seen that the maximum estimation errors occur in transients and, since the method used reduces stationary bias, it is the mean error that is of main interest.

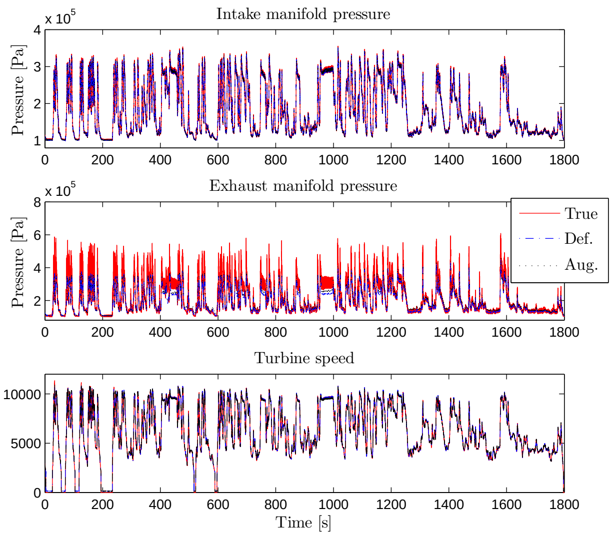

Fig. 3. State estimates using default and augmented observer applied to real measurement data.

7. CONCLUSIONS

A method for bias compensation in model based observers is developed. The idea is to find a low dimension augmentation of the model that describes the model biases. This augmented model is used to design an augmented observer, that results in a state estimate with reduced bias. A key result is a theoretical characterization of all possible bias augmentations.

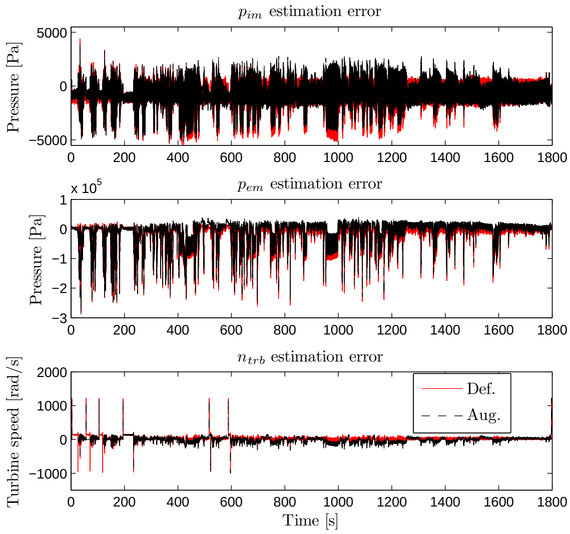

Fig. 4. Estimation errors using default and augmented observer applied to real measurement data.

The method is successfully applied to a diesel engine with variable geometry turbine (VGT) and exhaust gas recirculation (EGR), using a non-linear default model and input, and output data from an engine in a test cell. It is shown that an augmentation according to the suggested augmentation procedure reduces the mean estimation error, i.e. the bias.

Appendix A. ENGINE MODEL & DATA

The model, on which the method is applied, is a third order non-linear state space model of a six cylinder Scania diesel engine with VGT and EGR. The model states are intake and exhaust manifold pressures and turbine speed, and the inputs are injected amount of fuel, engine speed, VGT and EGR positions. It is based on a model developed in (Wahlström and Eriksson, 2006) but slightly simplified. The simplifications are that the states for the EGR mass fraction and actuator dynamics are removed.

The data is collected in collaboration with Scania. The data is from a six cylinder Scania diesel engine with VGT and EGR in a test cell and was collected during an European Transient Cycle (ETC).

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