

# Vehicle Propulsion Systems

## Lecture 7

### Non Electric Hybrid Propulsion Systems

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## Outline

### Repetition

### Short Term Storage

### Hybrid-Inertial Propulsion Systems

- Basic principles
- Design principles
- Modeling
- Continuously Variable Transmission

### Hybrid-Hydraulic Propulsion Systems

- Basics
- Modeling

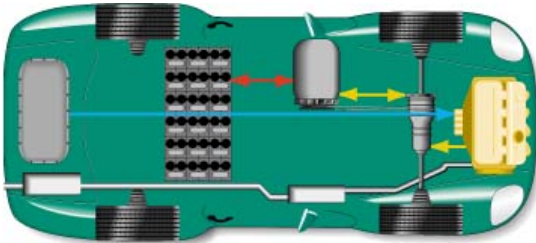
### Hydraulic Pumps and Motors

### Pneumatic Hybrid Engine Systems

### Case studies

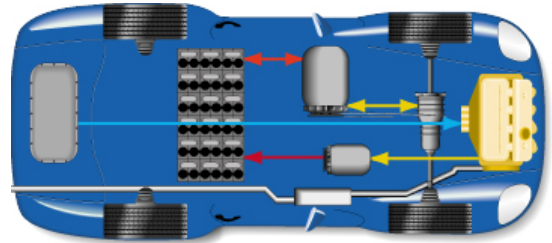
## Hybrid Electrical Vehicles – Parallel

- Two parallel energy paths
- One state in QSS framework, state of charge



## Hybrid Electrical Vehicles – Serial

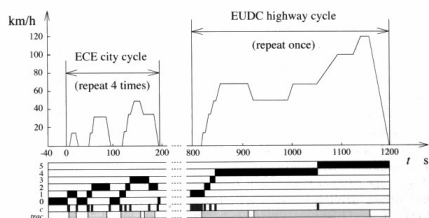
- Two paths working in parallel
- Decoupled through the battery
- Two states in QSS framework, state of charge & Engine speed



## Optimization, Optimal Control, Dynamic Programming

What gear ratios give the lowest fuel consumption for a given driving cycle?

– Problem presented in appendix 8.1 (and Lecture 3)



Problem characteristics

- Countable number of free variables,  $i_{g,j}, j \in [1, 5]$
- A “computable” criterion,  $m_f(\dots)$
- A “computable” set of constraints, model and cycle
- The formulated problem

$$\begin{aligned} \min_{i_{g,j}, j \in [1,5]} \quad & m_f(i_{g,1}, i_{g,2}, i_{g,3}, i_{g,4}, i_{g,5}) \\ \text{s.t.} \quad & \text{model and cycle is fulfilled} \end{aligned}$$

## General problem formulation

- Performance index

$$J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t) dt$$

- System model (constraints)

$$\frac{d}{dt}x = f(x(t), u(t), t), x(t_a) = x_a$$

- State and control constraints

$$u(t) \in U(t)$$

$$x(t) \in X(t)$$

## Optimal Control – Problem Motivation

Car with gas pedal  $u(t)$  as control input:

How to drive from A to B on a given time with minimum fuel consumption?

- Infinite dimensional decision variable  $u(t)$ .
- Criterion function  $\int_0^t \dot{m}_f(t) dt$
- Constraints:
  - Model of the car (the vehicle motion equation)

$$\begin{aligned} m_v \frac{d}{dt}v(t) &= F_r(v(t), u(t)) - (F_a(v(t)) + F_r(v(t)) + F_g(x(t))) \\ \frac{d}{dt}x(t) &= v(t) \\ \dot{m}_f &= f(v(t), u(t)) \end{aligned}$$

- Starting point  $x(0) = A$
- End point  $x(t_f) = B$
- Speed limits  $v(t) \leq g(x(t))$
- Limited control action  $0 \leq u(t) \leq 1$

## Dynamic programming – Problem Formulation

- Optimal control problem

$$\min J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t) dt$$

$$\text{s.t. } \frac{d}{dt}x = f(x(t), u(t), t)$$

$$x(t_a) = x_a$$

$$u(t) \in U(t)$$

$$x(t) \in X(t)$$

- $x(t), u(t)$  functions on  $t \in [t_a, t_b]$
- Search an approximation to the solution by discretizing
  - the state space  $x(t)$
  - and maybe the control signal  $u(t)$
 in both amplitude and time.
- The result is a combinatorial (network) problem

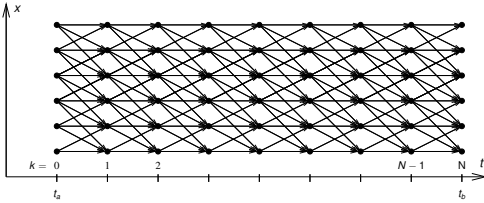
## Deterministic Dynamic Programming – Basic algorithm

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$

$$x_{k+1} = f_k(x_k, u_k)$$

Algorithm idea:

Start at the end and proceed backwards in time to evaluate the optimal cost-to-go and the corresponding control signal.



## Arc Cost Calculations

There are two ways for calculating the arc costs

- Calculate the exact control signal and cost for each arc.
  - Quasi-static approach
- Make a grid over the control signal and interpolate the cost for each arc.
  - Forward calculation approach

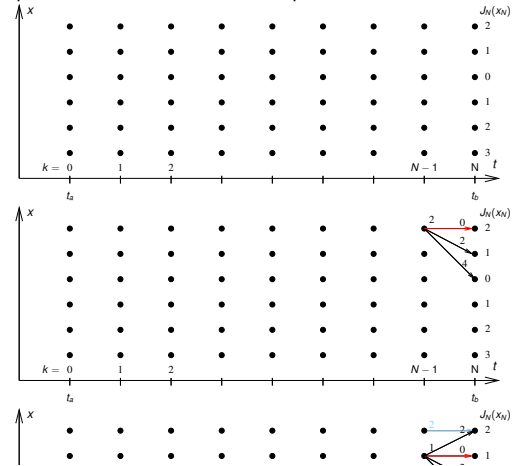
Matlab implementation – it is important to utilize matrix calculations

- Calculate the whole bundle of arcs in one step
- Add boundary and constraint checks

2D and 3D grid examples on whiteboard

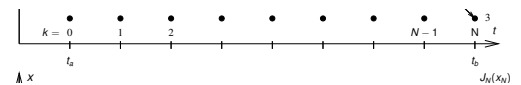
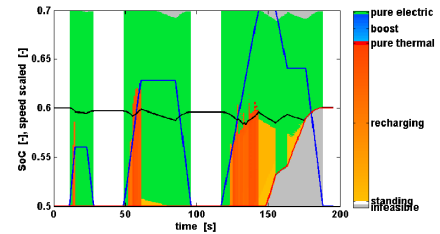
## Deterministic Dynamic Programming – Basic Algorithm

Graphical illustration of the solution procedure



## Parallel Hybrid Example

- Fuel-optimal torque split factor  $u(SOC, t) = \frac{T_{e-motor}}{T_{gearbox}}$
- ECE cycle
- Constraints  $SOC(t = t_f) \geq 0.6$ ,  $SOC \in [0.5, 0.7]$



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Repetition

### Short Term Storage

#### Hybrid-Inertial Propulsion Systems

- Basic principles
- Design principles
- Modeling
- Continuously Variable Transmission

#### Hybrid-Hydraulic Propulsion Systems

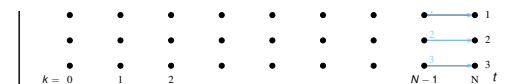
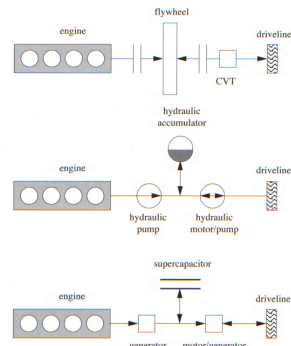
- Basics
- Modeling

#### Hydraulic Pumps and Motors

#### Pneumatic Hybrid Engine Systems

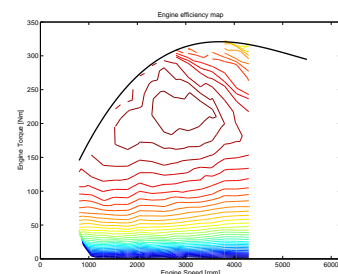
#### Case studies

## Examples of Short Term Storage Systems



## Basic Principles for Hybrid Systems

- Kinetic energy recovery
- Use “best” points – Duty cycle.
  - Run engine (fuel converter) at its optimal point.
  - Shut-off the engine.



## Short Term Storage – F1

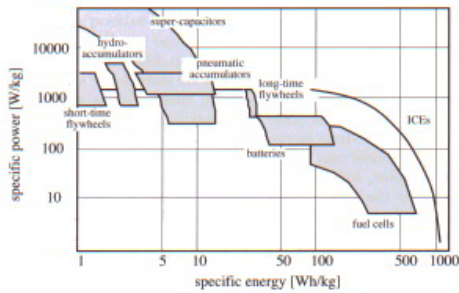
FIA allowed the usage of 60 kW, KERS (Kinetic Energy Recovery System) in F1 in 2009.

Technologies:

- Flywheel
- Batteries
- Super-Caps

## Power and Energy Densities

Asymptotic power and energy density – The Principle



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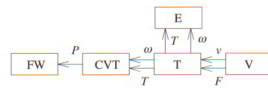
Hydraulic Pumps and Motors

Pneumatic Hybrid Engine Systems

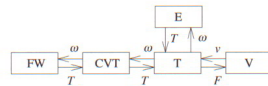
Case studies

## Causality for a hybrid-inertial propulsion system

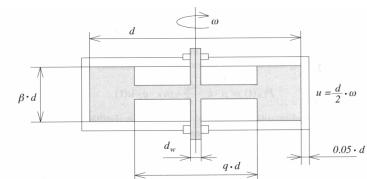
(a) quasistatic approach



(b) dynamic approach



## Flywheel accumulator



► Energy stored ( $\Theta_f = J_f$ ):

$$E_f = \frac{1}{2} \Theta_f \omega_f^2$$

► Wheel inertia

$$\Theta_f = \rho b \int_{Area} r^2 2\pi r dr = \dots = \frac{\pi}{2} \rho \frac{d^4}{16} (1 - q^4)$$

## Flywheel accumulator – Design principle

► Energy stored:

$$E_f = \frac{1}{2} \Theta_f \omega_f^2$$

► Wheel inertia

$$\Theta_f = \rho b \int_{Area} r^2 2\pi r dr = \dots = \frac{\pi}{2} \rho \frac{d^4}{16} (1 - q^4)$$

► Wheel Mass

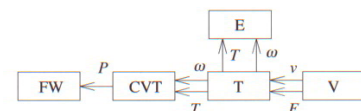
$$m_f = \pi \rho b d^2 (1 - q^2)$$

► Energy to mass ratio

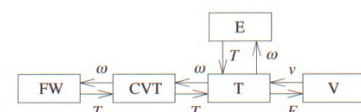
$$\frac{E_f}{m_f} = \frac{d^2}{16} (1 + q^2) \omega_f^2 = \frac{v^2}{4} (1 + q^2)$$

## Quasistatic Modeling of FW Accumulators

(a) quasistatic approach



(b) dynamic approach

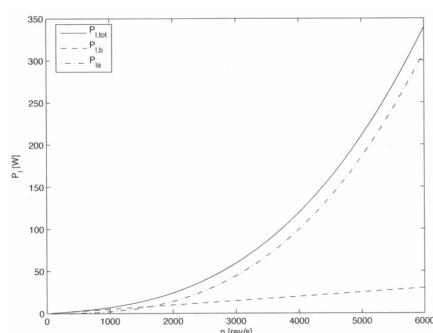


Basic equation for flywheel speed (SOC)

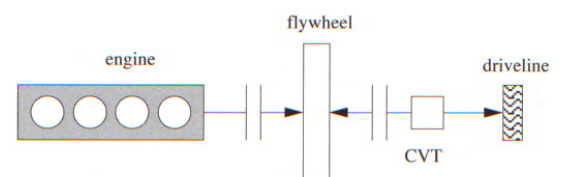
$$\Theta_f \omega_2(t) \frac{d}{dt} \omega_2(t) = -P_2(t) - P_1(t)$$

## Power losses as a function of speed

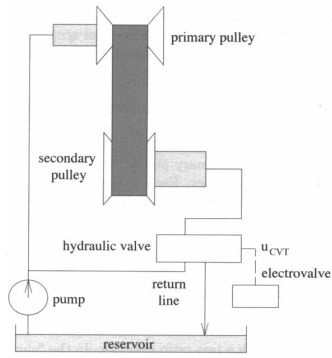
Air resistance and bearing losses



## Continuously Variable Transmission (CVT)



## CVT Principle



## CVT Modeling

- Transmission (gear) ratio  $\nu$ , speeds and transmitted torques

$$\omega_1(t) = \nu(t) \omega_2(t)$$

$$T_{t1}(t) = \nu (T_{t2}(t) - T_l(t))$$

- Newtons second law for the two pulleys

$$\Theta_1 \frac{d}{dt} \omega_1(t) = T_1(t) - T_{t1}(t)$$

$$\Theta_2 \frac{d}{dt} \omega_2(t) = T_2(t) - T_{t2}(t)$$

- System of equations give

$$T_1(t) = T_l(t) + \frac{T_2(t)}{\nu(t)} + \frac{\Theta_{CVT}(t)}{\nu(t)} \frac{d}{dt} \omega_2(t) + \Theta_1 \frac{d}{dt} \nu(t) \omega_2(t)$$

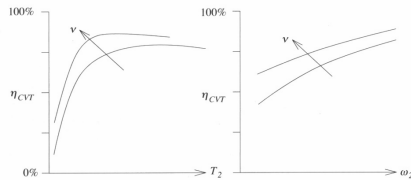
## CVT Modeling

- Transmission (gear) ratio  $\nu$ , speeds and transmitted torques

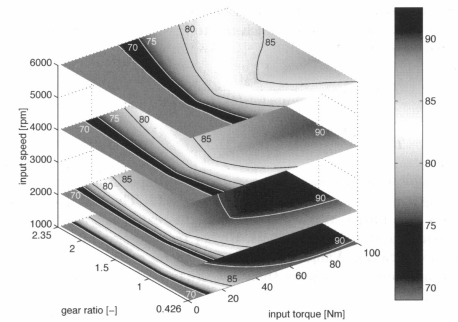
$$\omega_1(t) = \nu(t) \omega_2(t)$$

$$T_{t1}(t) = \nu (T_{t2}(t) - T_l(t))$$

- An alternative to model the losses, is to use an efficiency definition.



## Efficiencies for a Push-Belt CVT



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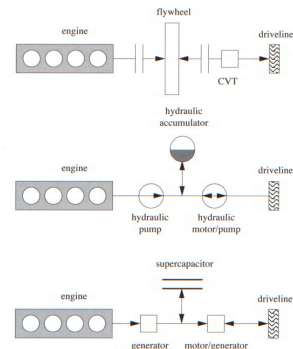
Basics  
Modeling

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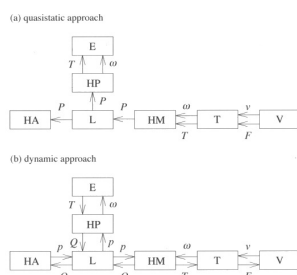
Pneumatic Hybrid Engine Systems

Case studies

## Examples of Short Term Storage Systems



## Causality for a hybrid-hydraulic propulsion system



## Modeling of a Hydraulic Accumulator

Modeling principle

–Energy balance

$$m_g c_v \frac{d}{dt} \theta_g(t) = -p \frac{d}{dt} V_g(t) - h A_w (\theta_g(t) - \theta_w)$$

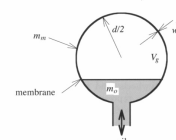
–Mass balance

(=volume for incompressible fluid)

$$\frac{d}{dt} V_g(t) = Q_2(t)$$

–Ideal gas law

$$p_g(t) = \frac{m_g R_g \theta_g(t)}{V_g(t)}$$



Power generation

$$P_2(t) = p_2(t) Q_2(t)$$

## Model Simplification

Simplifications made in thermodynamic equations to get a simple state equation.

- ▶ Assuming steady state conditions.
  - Eliminating  $\theta_g$  and the volume change gives

$$p_2(t) = \frac{h A_w \theta_w m_g R_g}{V_g(t) h A_w + m_g R_g Q_2(t)}$$

- ▶ Combining this with the power output gives

$$Q_2(t) = \frac{V_g(t)}{m_g} \frac{h A_w P_2(t)}{R_g \theta_w h A_w - R_g P_2(t)}$$

- ▶ Integrating  $Q_2(t)$  gives  $V_g$  as the state in the model.
- ▶ Modeling of the hydraulic systems efficiency, see the book.
- ▶ **A detail for the assignment**
  - This simplification can give problems in the simulation if parameter values are off. (Division by zero.)

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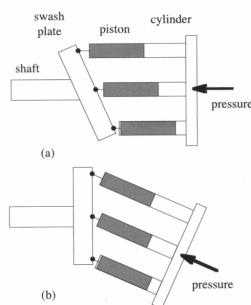
Modeling

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## Hydraulic Pumps



## Modeling of Hydraulic Motors

- ▶ Efficiency modeling

$$P_1(t) = \frac{P_2(t)}{\eta_{hm}(\omega_2(t), T_2(t))}, \quad P_2(t) > 0$$

$$P_1(t) = P_2(t) \eta_{hm}(\omega_2(t), -|T_2|(t)), \quad P_2(t) < 0$$

- ▶ Willans line modeling, describing the loss

$$P_1(t) = \frac{P_2(t) + P_0}{e}$$

- ▶ Physical modeling
  - Wilson's approach provided in the book.

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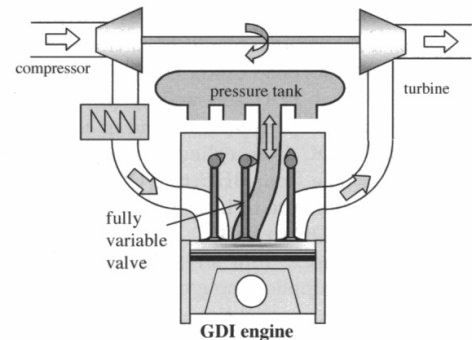
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## Pneumatic Hybrid Engine System

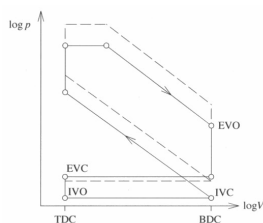


## Conventional SI Engine

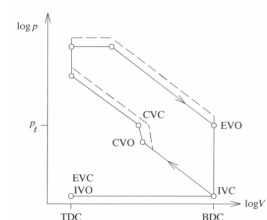
Compression and expansion model

$$p(t) = c v(t)^{-\gamma} \Rightarrow \log(p(t)) = \log(c) - \gamma \log(v(t))$$

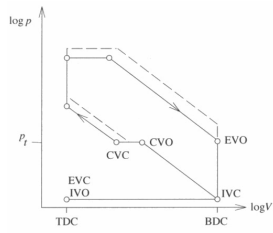
gives lines in the log-log diagram version of the pV-diagram



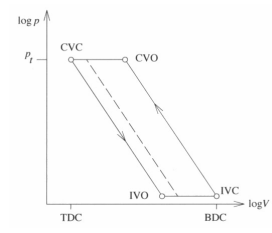
## Super Charged Mode



## Under Charged Mode



## Pneumatic Brake System



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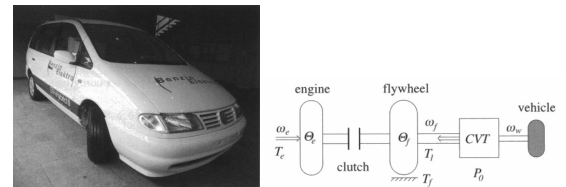
Hydraulic Pumps and Motors

Pneumatic Hybrid Engine Systems

Case studies

## Case Study 3: ICE and Flywheel Powertrain

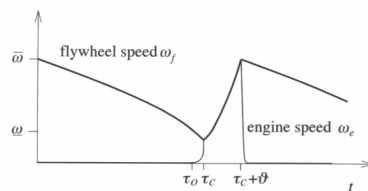
- Control of a ICE and Flywheel Powertrain
- Switching on and off engine



## Problem description

For each constant vehicle speed find the optimal limits for starting and stopping the engine

–Minimize fuel consumption



–Solved through parameter optimization ⇒ Map used for control

## Case Study 8: Hybrid Pneumatic Engine

- Local optimization of the engine thermodynamic cycle
- Different modes to select between
- Dynamic programming of the mode selection

