Vehicle Propulsion Systems
Lecture 09
Case Study 6 Fuel Cell Vehicle and Optimal Control

Lars Eriksson
Associate Professor (Docent)

Vehicular Systems
Linköping University

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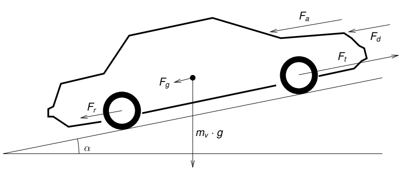
Outline

- Repetition
- Case study 6: Fuel Optimal Trajectories of a Racing FCEV
  - Model compilation
- Model simplification
- Formulating the optimal control problem
- Optimal Controllers Solutions
- Some Additional Material – Fuel Consumption

The Vehicle Motion Equation

Newtons second law for a vehicle

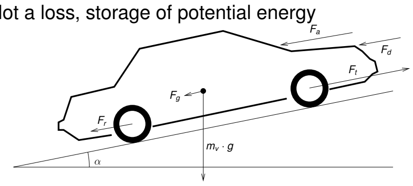
$$m_v \frac{d}{dt} v(t) = F_t(t) - (F_a(t) + F_r(t) + F_g(t) + F_d(t))$$



- F_t – tractive force
- F_a – aerodynamic drag force
- F_r – rolling resistance force
- F_g – gravitational force
- F_d – disturbance force

Gravitational Force

- Gravitational load force
  –Not a loss, storage of potential energy



- Up- and down-hill driving produces forces.

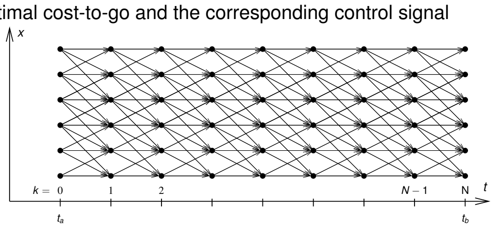
$$F_g = m_v \, g \, \sin(\alpha)$$

- Flat road assumed $\alpha = 0$ if nothing else is stated (In the book).

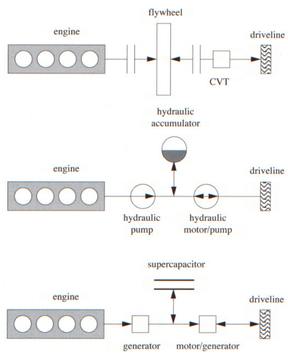
Deterministic Dynamic Programming – Basic algorithm

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$
$$x_{k+1} = f_k(x_k, u_k)$$

Algorithm idea:
Start at the end and proceed backward in time to evaluate the optimal cost-to-go and the corresponding control signal

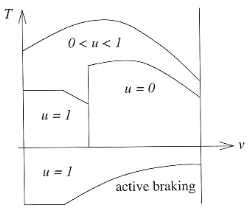


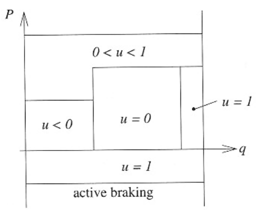
Examples of Short Term Storage Systems



Heuristic Control Approaches

- Parallel hybrid vehicle (electric assist)

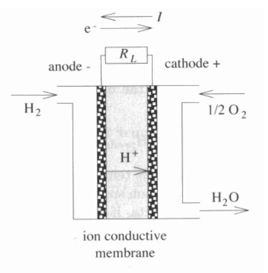


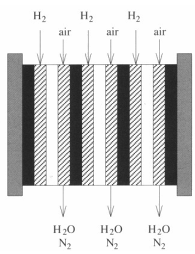


- Determine control output as function of some selected state variables:
  vehicle speed, engine speed, state of charge, power demand, motor speed, temperature, vehicle acceleration, torque demand

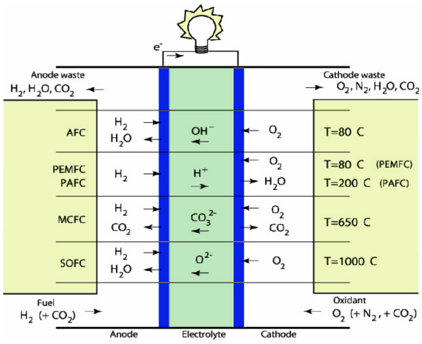
Fuel Cell Basic Principles

- Convert fuel directly to electrical energy
- Let an ion pass from an anode to a cathode
- Take out electrical work from the electrons
- Fuel cells are stacked ($U_{cell} \leq 1$V)





Overview of Different Fuel Cell Technologies

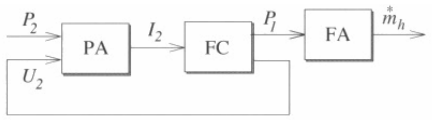


Hydrogen Fuel Storage

- Hydrogen storage is problematic – Challenging task.
- Some examples of different options.
  - High pressure bottles
  - Liquid phase – Cryogenic storage, -253°C.
  - Metal hydride
  - Sodium borohydride $NaBH_4$

Quasistatic Modeling of a Fuel Cell

- Causality diagram



- Power amplifier (Current controller)
- Fuel amplifier (Fuel controller)
- Standard modeling approach

Fuel Cell Thermodynamics

- Starting point reaction equation

$$H_2 + \frac{1}{2} O_2 \Rightarrow 2\, H_20$$

- Open system energy – Enthalpy H

$$H = U + pV$$

- Reversible energy – Gibbs free energy G

$$G = H + TS$$

- Open circuit cell voltages

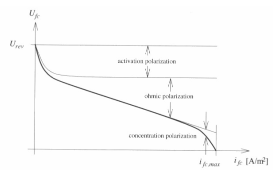
$$U_{rev} = -\frac{\Delta G}{n_e\, F}, \qquad U_{id} = -\frac{\Delta H}{n_e\, F}, \qquad U_{rev} = \eta_{id}\, U_{id}$$

  F – Faradays constant ($F = q\, N_0$)
- Under load

$$P_l = I_{fc}(t)\,(U_{id} - U_{fc}(t))$$

Fuel Cell Performance – Polarization curve

- Polarization curve of a fuel cell
  Relating current density $i_{fc}(t) = I_{fc}(t)/A_{fc}$, and cell voltage $U_{fc}(t)$



  Curve for one operating condition
- Fundamentally different compared to combustion engine/electrical motor
- Excellent part load behavior
  –When considering only the cell

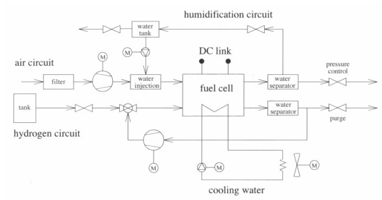
Fuel Cell System Modeling

- Describe all subsystems with models

$$P_2(t) = P_{st}(t) - P_{aux}(t)$$

$$P_{aux} = P_0 + P_{em}(t) + P_{ahp}(t) + P_{hp}(t) + P_{cl}(t) + P_{cf}(t)$$

em–electric motor, ahp – humidifier pump, hp – hydrogen recirculation pump, cl – coolant pump, cf – cooling fan.



- Submodels for: Hydrogen circuit, air circuit, water circuit, and coolant circuit

Outline

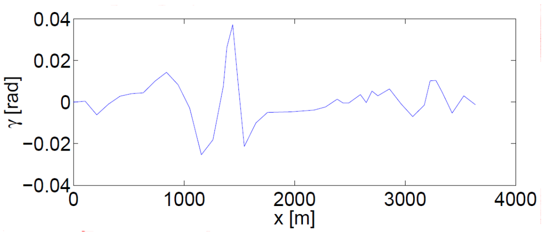
Problem Setup

- Run a fuel cell vehicle optimally on a racetrack

  - Start up lap
  - Repeated runs on the track
- Path to the solution
  - Measurements – Model
  - Simplified model
  - Optimal control solutions

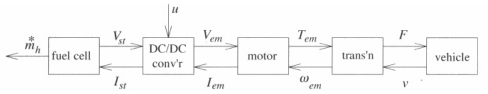
Problem Setup – Road Slope Given

Road slope $\gamma = \alpha(x)$

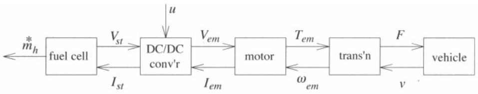


Model Causality

Model causality – Dynamic model



Model Component – Fuel Cell



- Current in the cell and losses

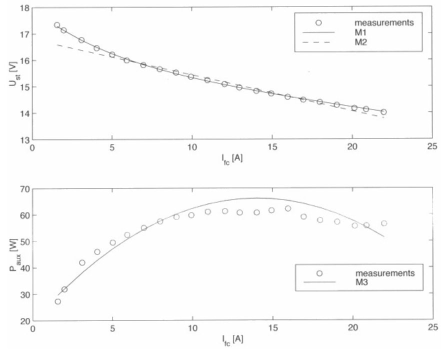
$$I_{fc}(t) = I_{fc}(t) + I_{aux}(t)$$

- Current and hydrogen flow

$$\dot{m}_H(t) = c_9\, I_{fc}(t)$$

- Next step: Polarization curve and auxiliary consumption

Fuel Cell – Polarization and Auxiliary Components



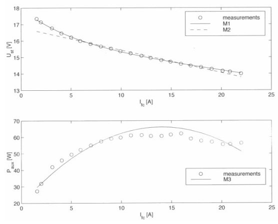
Fuel Cell – Polarization and Auxiliary Components

- Polarization curve

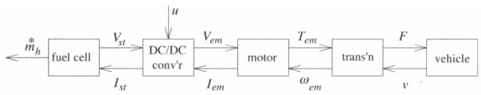
$$U_{st}(t) = c_0 + c_1 \cdot e^{-c_2 \cdot I_{fc}(t)} - c_3 \cdot I_{fc}(t)$$

- Auxiliary power

$$P_{aux}(t) = c_6 + c_7 \cdot I_{fc}(t) + c_8 \cdot I_{fc}(t)^2$$



Model Component – DC Motor Controller



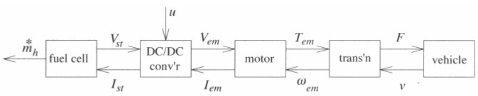
- DC motor voltage (from control signal u)

$$U_{em}(t) = \kappa\, \omega_{em}(t) + K\, R_{em}\, u(t)$$

- Current requirement at the stack

$$I_{st} = \frac{U_{em}(t) I_{em}(t)}{\eta_c\, U_{st}(t)}$$

Model Component – DC Motor



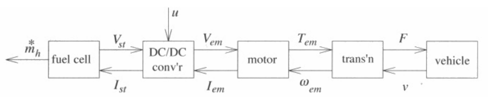
- DC motor current

$$I_{em}(t) = \frac{U_{em}(t) - \kappa\, \omega_{em}(t)}{R_{em}}$$

- DC motor torque

$$T_{em}(t) = \kappa_{em}\, I_{em}(t)$$

Model Component – Transmission and Wheels



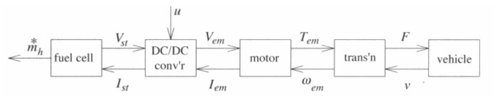
- Tractive force

$$F(t) = \eta_t^{\pm 1}\, \frac{\gamma\, T_{em}(t)}{r_w}$$

- Rotational speed

$$\omega_{em}(t) = \frac{\gamma\, v(t)}{r_w}$$

Model Compilation 1 – Vehicle



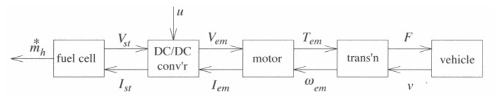
- The vehicle tractive force can now be expressed as

$$F(t) = \frac{\eta_t \gamma}{r_w} \kappa_{em} K \, u(t)$$

- Dynamic vehicle velocity and position model

$$\frac{d}{dt} v(t) = h_1 \, u(t) - h_2 \, v^2(t) - g_0 - g_1 \, \alpha(x(t))$$
$$\frac{d}{dt} x(t) = v(t)$$

Model Compilation 2 – Fuel Consumption



- Fuel flow, $\dot{m}_H(t) = c_9 \, I_{fc}(t)$

$$I_{fc}(t) = \frac{P_{aux}(I_{st}(t))}{U_{st}(I_{st}(t))} + \frac{K \, u(t)}{\eta_c \, U_{st}(I_{st}(t))} \left(K \, R_{em} \, u(t) + \kappa_{em} \frac{\gamma}{r_w} \, v(t) \right)$$

  –Implicit nonlinear static function
- Simpler model

$$\dot{m}_H(t) = b_0 + b_1 \, v(t) \, u(t) + b_2 \, u^2(t)$$

Outline

Repetition

Case study 6: Fuel Optimal Trajectories of a Racing FCEV
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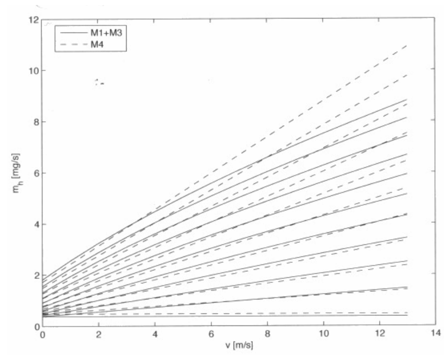
Model simplification

Formulating the optimal control problem

Optimal Controllers Solutions

Some Additional Material – Fuel Consumption

Simplified Fuel Consumption – Validation



Detour

- Occam's razor:
  –The explanation of any phenomenon should make as few assumptions as possible.
  Shave of those who are unnecessary.
- Law of Parsimony: Among others a factor in statistics: In general, mathematical models with the smallest number of parameters are preferred as each parameter introduced into the model adds some uncertainty to it.
- Another viewpoint.
  Try to simplify the problem you solve as much as possible.
  –Neglect effects and be proud when you are successful!

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Optimal Control Problems

- Start of the cycle

$$v(0) = 0, \qquad x(0) = 0$$
$$\lambda_1(t_f) = 0, \qquad x(t_f) = x_f = v_m \, t_f$$

- Periodic route

$$x(0) = 0$$
$$\lambda_1(t_f) = \lambda_1(0), \qquad x(t_f) = x_f = v_m \, t_f, \qquad v(t_f) = v(0)$$

PID Cruise Controller – Baseline for Comparison

Simple controller for the start

$$u(t) = K_p \, (f \, v_m - v(t)) + K_i \int_0^t (f \, v_m - v_t(t)) dt$$

f-tuning parameter ≈ 1 to allow for matching the average speed

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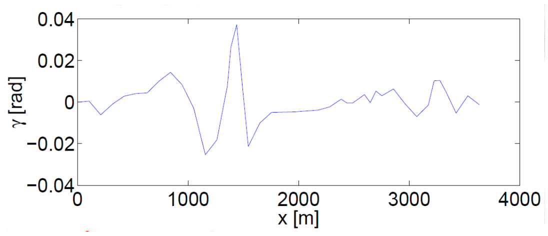
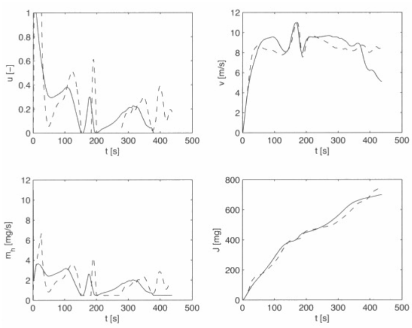
Road slope $\gamma = \alpha(x)$



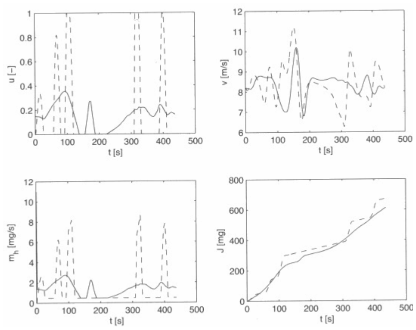
Fuel Optimal Trajectory – Start

Fuel optimal trajectory has 7% lower fuel consumption



Fuel Optimal Trajectory – Continuous Driving

Fuel optimal trajectory has 9% lower fuel consumption



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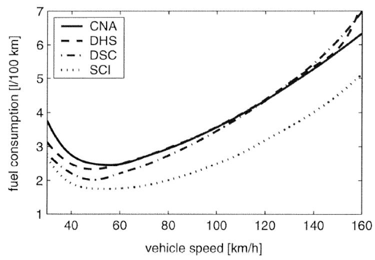
Fuel Optimal Speed for Normal Driving

ICE vehicle (light weight 800 kg)



Engine Map and Gearbox Layout

CI engine (light weight 800 kg)