

Vehicle Propulsion Systems

Lecture 9

Case Study 6 Fuel Cell Vehicle and Optimal Control

Lars Eriksson
Associate Professor (Docent)

Vehicular Systems
Linköping University

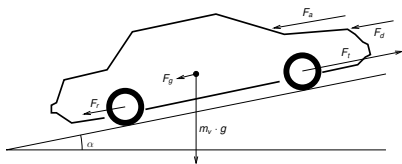
December 7, 2014

1/40

The Vehicle Motion Equation

Newtons second law for a vehicle

$$m_v \frac{d}{dt} v(t) = F_t(t) - (F_a(t) + F_r(t) + F_g(t) + F_d(t))$$



- ▶ F_t – tractive force
- ▶ F_a – aerodynamic drag force
- ▶ F_r – rolling resistance force
- ▶ F_g – gravitational force
- ▶ F_d – disturbance force

3/40

Outline

Repetition

Case study 6: Fuel Optimal Trajectories of a Racing FCEV
Model compilation

Model simplification

Formulating the optimal control problem

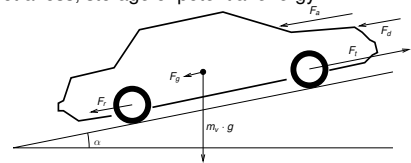
Optimal Controllers Solutions

Some Additional Material – Fuel Consumption

2/40

Gravitational Force

- ▶ Gravitational load force
–Not a loss, storage of potential energy



- ▶ Up- and down-hill driving produces forces.

$$F_g = m_v g \sin(\alpha)$$

- ▶ Flat road assumed $\alpha = 0$ if nothing else is stated (In the book).

4/40

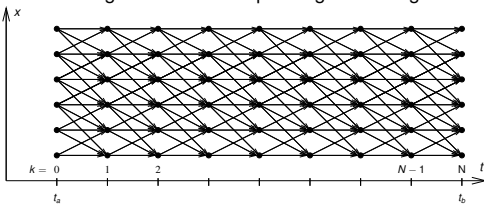
Deterministic Dynamic Programming – Basic algorithm

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$

$$x_{k+1} = f_k(x_k, u_k)$$

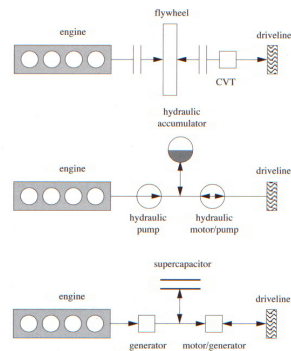
Algorithm idea:

Start at the end and proceed backward in time to evaluate the optimal cost-to-go and the corresponding control signal



5/40

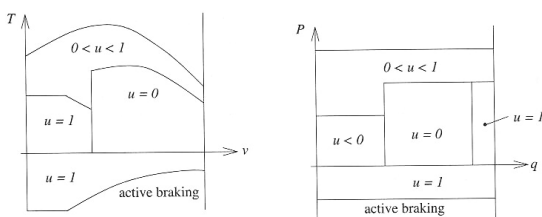
Examples of Short Term Storage Systems



6/40

Heuristic Control Approaches

- ▶ Parallel hybrid vehicle (electric assist)

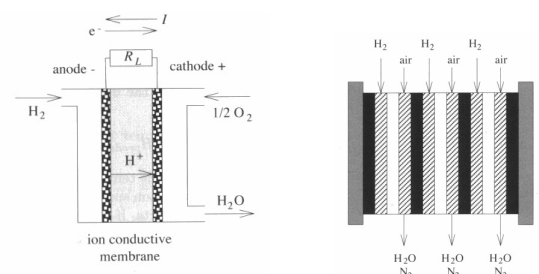


- ▶ Determine control output as function of some selected state variables:
vehicle speed, engine speed, state of charge, power demand, motor speed, temperature, vehicle acceleration, torque demand

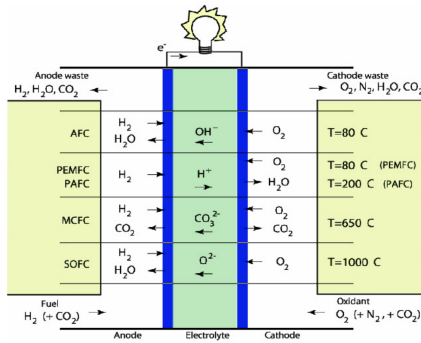
7/40

Fuel Cell Basic Principles

- ▶ Convert fuel directly to electrical energy
- ▶ Let an ion pass from an anode to a cathode
- ▶ Take out electrical work from the electrons
- ▶ Fuel cells are stacked ($U_{cell} \leq 1V$)



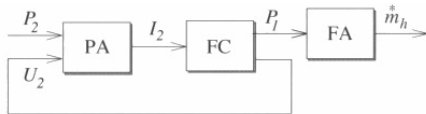
8/40



- ▶ Hydrogen storage is problematic – Challenging task.
- ▶ Some examples of different options.
 - ▶ High pressure bottles
 - ▶ Liquid phase – Cryogenic storage, -253°C .
 - ▶ Metal hydride
 - ▶ Sodium borohydride $NaBH_4$

Quasistatic Modeling of a Fuel Cell

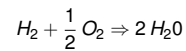
- ▶ Causality diagram



- ▶ Power amplifier (Current controller)
- ▶ Fuel amplifier (Fuel controller)
- ▶ Standard modeling approach

Fuel Cell Thermodynamics

- ▶ Starting point reaction equation



- ▶ Open system energy – Enthalpy H

$$H = U + pV$$

- ▶ Reversible energy – Gibbs free energy G

$$G = H + TS$$

- ▶ Open circuit cell voltages

$$U_{rev} = -\frac{\Delta G}{n_e F}, \quad U_{id} = -\frac{\Delta H}{n_e F}, \quad U_{rev} = \eta_{id} U_{id}$$

F – Faradays constant ($F = q N_0$)

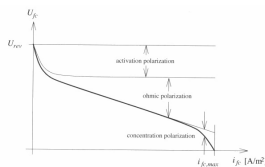
- ▶ Under load

$$P_I = I_{fc}(t) (U_{id} - U_{fc}(t))$$

Fuel Cell Performance – Polarization curve

- ▶ Polarization curve of a fuel cell

Relating current density $i_{fc}(t) = I_{fc}(t)/A_{fc}$, and cell voltage $U_{fc}(t)$



Curve for one operating condition

- ▶ Fundamentally different compared to combustion engine/electrical motor
- ▶ Excellent part load behavior – When considering only the cell

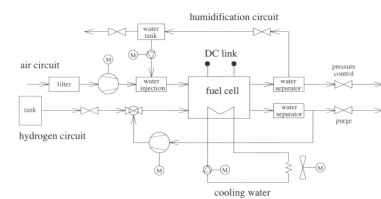
Fuel Cell System Modeling

- ▶ Describe all subsystems with models

$$P_2(t) = P_{st}(t) - P_{aux}(t)$$

$$P_{aux} = P_0 + P_{em}(t) + P_{ahp}(t) + P_{hp}(t) + P_{cl}(t) + P_{cf}(t)$$

em – electric motor, ahp – humidifier pump, hp – hydrogen recirculation pump, cl – coolant pump, cf – cooling fan.



- ▶ Submodels for: Hydrogen circuit, air circuit, water circuit, and coolant circuit

Outline

Repetition

Case study 6: Fuel Optimal Trajectories of a Racing FCEV

Model compilation

Model simplification

Formulating the optimal control problem

Optimal Controllers Solutions

Some Additional Material – Fuel Consumption

Problem Setup

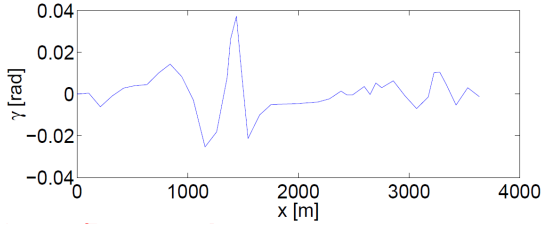
- ▶ Run a fuel cell vehicle optimally on a racetrack



- ▶ Start up lap
- ▶ Repeated runs on the track
- ▶ Path to the solution
 - ▶ Measurements – Model
 - ▶ Simplified model
 - ▶ Optimal control solutions

Problem Setup – Road Slope Given

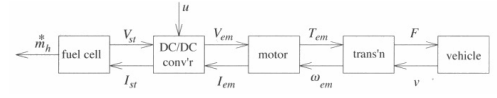
Road slope $\gamma = \alpha(x)$



17/40

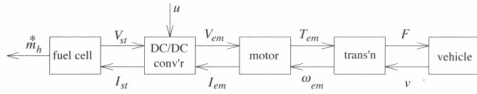
Model Causality

Model causality – Dynamic model



18/40

Model Component – Fuel Cell



- Current in the cell and losses

$$I_{fc}(t) = I_c(t) + I_{aux}(t)$$

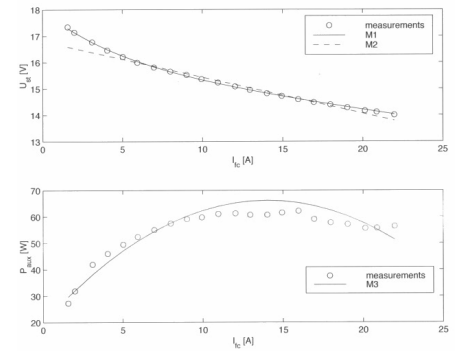
- Current and hydrogen flow

$$\dot{m}_H(t) = c_9 I_{fc}(t)$$

- Next step: Polarization curve and auxiliary consumption

19/40

Fuel Cell – Polarization and Auxiliary Components



20/40

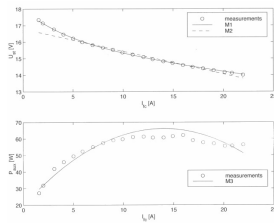
Fuel Cell – Polarization and Auxiliary Components

- Polarization curve

$$U_{st}(t) = c_0 + c_1 \cdot e^{-c_2 \cdot I_{fc}(t)} - c_3 \cdot I_{fc}(t)$$

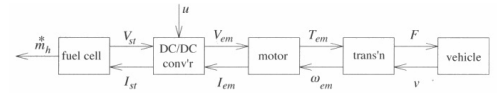
- Auxiliary power

$$P_{aux}(t) = c_6 + c_7 \cdot I_{fc}(t) + c_8 \cdot I_{fc}(t)^2$$



21/40

Model Component – DC Motor Controller



- DC motor voltage (from control signal u)

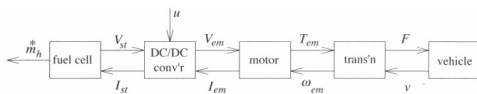
$$U_{em}(t) = \kappa \omega_{em}(t) + K R_{em} u(t)$$

- Current requirement at the stack

$$I_{st} = \frac{U_{em}(t) I_{em}(t)}{\eta_c U_{st}(t)}$$

22/40

Model Component – DC Motor



- DC motor current

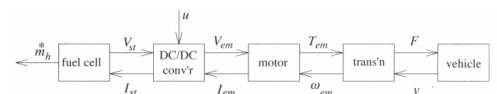
$$I_{em}(t) = \frac{U_{em}(t) - \kappa \omega_{em}(t)}{R_{em}}$$

- DC motor torque

$$T_{em}(t) = \kappa_{em} I_{em}(t)$$

23/40

Model Component – Transmission and Wheels



- Tractive force

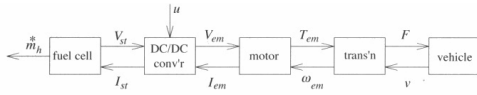
$$F(t) = \eta_t^{\pm 1} \frac{\gamma T_{em}(t)}{r_w}$$

- Rotational speed

$$\omega_{em}(t) = \frac{\gamma v(t)}{r_w}$$

24/40

Model Compilation 1 – Vehicle



- The vehicle tractive force can now be expressed as

$$F(t) = \frac{\eta_t \gamma}{r_w} \kappa_{em} K u(t)$$

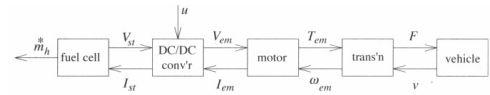
- Dynamic vehicle velocity and position model

$$\frac{d}{dt} v(t) = h_1 u(t) - h_2 v^2(t) - g_0 - g_1 \alpha(x(t))$$

$$\frac{d}{dt} x(t) = v(t)$$

25/40

Model Compilation 2 – Fuel Consumption



- Fuel flow, $\dot{m}_H(t) = c_9 I_{fc}(t)$

$$I_{fc}(t) = \frac{P_{aux}(I_{st}(t))}{U_{st}(I_{st}(t))} + \frac{K u(t)}{\eta_c U_{st}(I_{st}(t))} \left(K R_{em} u(t) + \kappa_{em} \frac{\gamma}{r_w} v(t) \right)$$

–Implicit nonlinear static function

- Simpler model

$$\dot{m}_H(t) = b_0 + b_1 v(t) u(t) + b_2 u^2(t)$$

26/40

Outline

Repetition

Case study 6: Fuel Optimal Trajectories of a Racing FCEV
Model compilation

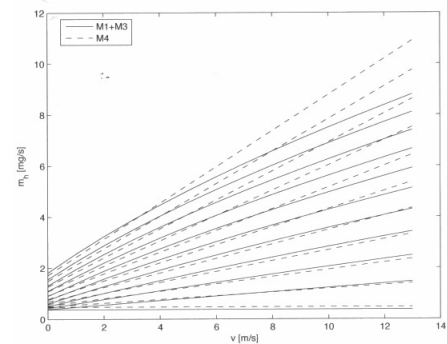
Model simplification

Formulating the optimal control problem

Optimal Controllers Solutions

Some Additional Material – Fuel Consumption

Simplified Fuel Consumption – Validation



27/40

28/40

Detour

- Occam's razor:
 - The explanation of any phenomenon should make as few assumptions as possible.
 - Shave of those who are unnecessary.
- Law of Parsimony: Among others a factor in statistics: In general, mathematical models with the smallest number of parameters are preferred as each parameter introduced into the model adds some uncertainty to it.
- Another viewpoint.
 - Try to simplify the problem you solve as much as possible.
 - Neglect effects and be proud when you are successful!

29/40

Outline

Repetition

Case study 6: Fuel Optimal Trajectories of a Racing FCEV
Model compilation

Model simplification

Formulating the optimal control problem

Optimal Controllers Solutions

Some Additional Material – Fuel Consumption

30/40

Optimal Control Problems

- Start of the cycle

$$v(0) = 0, \quad x(0) = 0$$

$$\lambda_1(t_f) = 0, \quad x(t_f) = x_f = v_m t_f$$

- Periodic route

$$x(0) = 0$$

$$\lambda_1(t_f) = \lambda_1(0), \quad x(t_f) = x_f = v_m t_f, \quad v(t_f) = v(0)$$

31/40

PID Cruise Controller – Baseline for Comparison

Simple controller for the start

$$u(t) = K_p (f v_m - v(t)) + K_i \int_0^t (f v_m - v_i(t)) dt$$

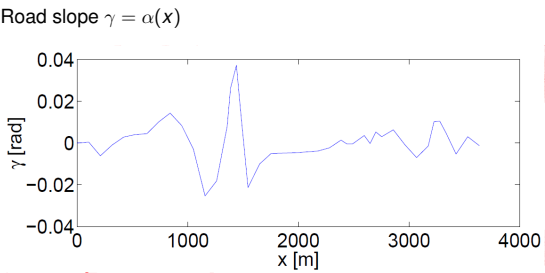
f -tuning parameter ≈ 1 to allow for matching the average speed

32/40

Outline

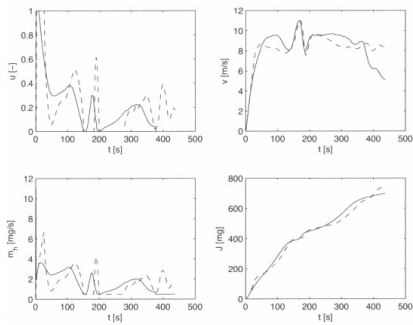
- Repetition
- Case study 6: Fuel Optimal Trajectories of a Racing FCEV
 - Model compilation
 - Model simplification
 - Formulating the optimal control problem
- Optimal Controllers Solutions
- Some Additional Material – Fuel Consumption

Problem Setup – Road Slope Given



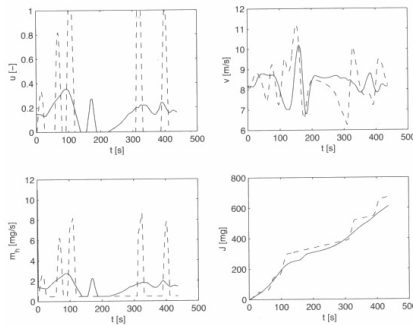
Fuel Optimal Trajectory – Start

Fuel optimal trajectory has 7% lower fuel consumption



Fuel Optimal Trajectory – Continuous Driving

Fuel optimal trajectory has 9% lower fuel consumption

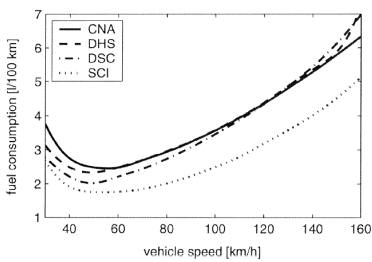


Outline

- Repetition
- Case study 6: Fuel Optimal Trajectories of a Racing FCEV
 - Model compilation
 - Model simplification
 - Formulating the optimal control problem
- Optimal Controllers Solutions
- Some Additional Material – Fuel Consumption

Fuel Optimal Speed for Normal Driving

ICE vehicle (light weight 800 kg)



Engine Map and Gearbox Layout

CI engine (light weight 800 kg)

