

# Vehicle Propulsion Systems

## Lecture 5

### Deterministic Dynamic Programming and Some Examples

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## Outline

### Repetition

#### "Traditional" Optimization

Problem motivation  
Different Classes of Problems

### Optimal Control

Problem Motivation

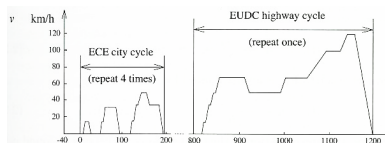
### Deterministic Dynamic Programming

Problem setup and basic solution idea  
Cost Calculation – Two Implementation Alternatives  
The Provided Tools

### Case Studies

Energy Management of a Parallel Hybrid

## Energy consumption for cycles



Numerical values for MVEG-95, ECE, EUDC

$$\text{air drag} = \frac{1}{x_{\text{tot}}} \sum_{i \in \text{trac}} \bar{v}_i^3 h = \{319, 82.9, 455\}$$

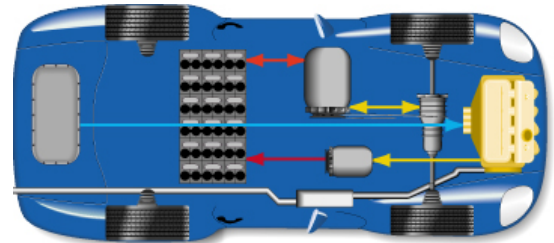
$$\text{rolling resistance} = \frac{1}{x_{\text{tot}}} \sum_{i \in \text{trac}} \bar{v}_i h = \{.856, 0.81, 0.88\}$$

$$\text{kinetic energy} = \frac{1}{x_{\text{tot}}} \sum_{i \in \text{trac}} \bar{a}_i \bar{v}_i h = \{0.101, 0.126, 0.086\}$$

$$\bar{E}_{\text{MVEG-95}} \approx A_f c_d 1.9 \cdot 10^4 + m_v c_r 8.4 \cdot 10^2 + m_v 10 \quad \text{kJ/100km}$$

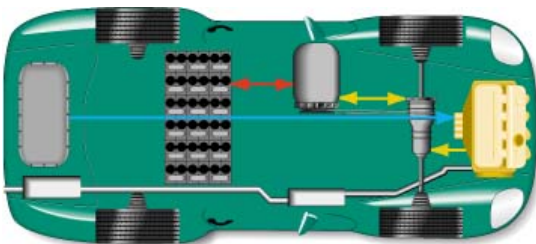
## Hybrid Electrical Vehicles – Serial

- Two paths working in parallel
- Decoupled through the battery



## Hybrid Electrical Vehicles – Parallel

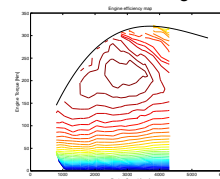
- Two parallel energy paths



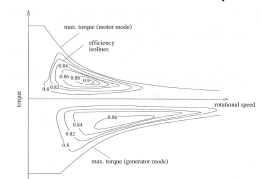
## Component modeling

- Model energy (power) transfer and losses
- Using maps  $\eta = f(T, \omega)$

### Combustion engine map



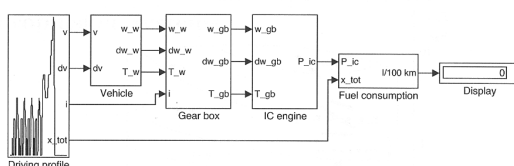
### Electric motor map



- Using parameterized (scalable) models –Willans approach

## Model implemented in QSS

### Conventional powertrain



Efficient computations are important  
–For example if we want to do optimization and sensitivity studies.

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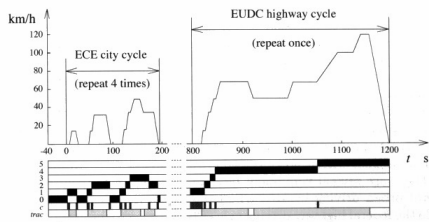
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## Problem motivation

What gear ratios give the lowest fuel consumption for a given driving cycle?

– Problem presented in appendix 8.1



Problem characteristics

- ▶ Countable number of free variables,  $i_{g,j}, j \in [1, 5]$
- ▶ A “computable” cost,  $m_f(\dots)$
- ▶ A “computable” set of constraints, model and cycle
- ▶ The formulated problem

$$\begin{aligned} \min_{i_{g,j}, j \in [1,5]} \quad & m_f(i_{g,1}, i_{g,2}, i_{g,3}, i_{g,4}, i_{g,5}) \\ \text{s.t.} \quad & \text{model and cycle is fulfilled} \end{aligned}$$

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## Optimization – Linear Programming

- ▶ Linear problem

$$\begin{aligned} \min_x \quad & c^T x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

- ▶ Convex problem
- ▶ Much analyzed: existence, uniqueness, sensitivity
- ▶ Many algorithms: Simplex the most famous

- ▶ About the word *Programming*

– The solution to a problem was called a program

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## Optimization – Non-Linear Programming

- ▶ Non-linear problem

$$\begin{aligned} \min_x \quad & f(x) \\ \text{s.t.} \quad & g(x) = 0 \\ & x \geq 0 \end{aligned}$$

- ▶ For convex problems
  - Much analyzed: existence, uniqueness, sensitivity
  - Many algorithms
- ▶ For non-convex problems
  - Some special problems have solutions
  - Local optimum is not necessarily a global optimum

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## Mixed Integer and Combinatorial Optimization

- ▶ Problem

$$\begin{aligned} \min_{x,y} \quad & f(x, y) \\ \text{s.t.} \quad & g(x, y) = 0 \\ & x \geq 0 \\ & y \in \mathbb{Z}^+ \end{aligned}$$

- ▶ Inherently non-convex  $y$ 
  - Generally hard problems to solve.
- ▶ Much analyzed
  - Existence, uniqueness, sensitivity
  - Many types of problems
  - Many different algorithms

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## Some comments on problem solver

- ▶ Find the “right” problem formulation
- ▶ Use the right solver for the problem

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## Optimal Control – Problem Motivation

Car with gas pedal  $u(t)$  as control input:

How to drive from A to B on a given time with minimum fuel consumption?

- ▶ Infinite dimensional decision variable  $u(t)$ .
- ▶ Cost function  $\int_0^t \dot{m}_f(t) dt$
- ▶ Constraints:
  - ▶ Model of the car (the vehicle motion equation)

$$\begin{aligned} m_v \frac{d}{dt} v(t) &= F_t(v(t), u(t)) - (F_a(v(t)) + F_r(v(t)) + F_g(x(t))) \\ \frac{d}{dt} x(t) &= v(t) \\ \dot{m}_f &= f(v(t), u(t)) \end{aligned}$$

- ▶ Starting point  $x(0) = A$
- ▶ End point  $x(t_f) = B$
- ▶ Speed limits  $v(t) \leq g(x(t))$
- ▶ Limited control action  $0 \leq u(t) \leq 1$
- ▶ Difficult (impossible) problem to solve analytically

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## General problem formulation

- ▶ Performance index

$$J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t) dt$$

- ▶ System model (constraints)

$$\frac{d}{dt} x = f(x(t), u(t), t), x(t_a) = x_a$$

- ▶ State and control constraints

$$u(t) \in U(t)$$

$$x(t) \in X(t)$$

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- ▶ Old subject
- ▶ Rich theory
  - ▶ Old theory from calculus of variations
  - ▶ Much theory and many methods were developed during 50's-70's
  - ▶ Theory and methods are still being actively developed
- ▶ Dynamic programming, Richard Bellman, 50's.
- ▶ A modern success story:
  - Model predictive control (MPC)
- ▶ Now a new interest for collocation methods:
  - A few during 1990's
  - Much interest 2000–

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## Dynamic programming – Problem Formulation

- ▶ Optimal control problem

$$\begin{aligned} \min J(u) &= \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t) dt \\ \text{s.t. } \frac{d}{dt}x &= f(x(t), u(t), t) \\ x(t_a) &= x_a \\ u(t) &\in U(t) \\ x(t) &\in X(t) \end{aligned}$$

- ▶  $x(t)$ ,  $u(t)$  functions on  $t \in [t_a, t_b]$
- ▶ Search an approximation to the solution by discretizing
  - ▶ the state space  $x(t)$
  - ▶ and maybe the control signal  $u(t)$
 in both amplitude and time.
- ▶ The result is a combinatorial (network) problem

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## Dynamic Programming (DP) – Problem Formulation

- ▶ Find the optimal control sequence  $\pi^0(x_0) = \{u_0, u_1, \dots, u_{N-1}\}$  minimizing:

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, w_k)$$

- ▶ subject to:

$$\begin{aligned} x_{k+1} &= f_k(x_k, u_k, w_k) \\ x_0 &= x(t=0) \\ x_k &\in X_k \\ u_k &\in U_k \end{aligned}$$

- ▶ Disturbance  $w_k$
- ▶ Stochastic vs Deterministic DP

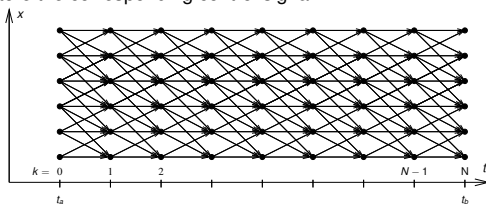
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## DDP – Basic Algorithm

$$\begin{aligned} J(x_0) &= g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k) \\ x_{k+1} &= f_k(x_k, u_k) \end{aligned}$$

### Bellman's Theory and Algorithm:

- Start at the end and proceed backward in time
- Determine the optimal cost-to-go
- Store the corresponding control signal



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## DDP – Basic algorithm

$$\begin{aligned} J(x_0) &= g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k) \\ x_{k+1} &= f_k(x_k, u_k) \end{aligned}$$

### Algorithm:

1. Set  $k = N$ , and assign final cost  $J_N(x_N) = g_N(x_N)$
2. Set  $k = k - 1$
3. For all points in the state-space grid, find the optimal cost to go

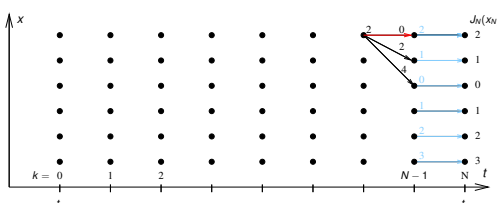
$$J_k(x_k) = \min_{u_k \in U_k(x_k)} g_k(x_k, u_k) + J_{k+1}(f_k(x_k, u_k))$$

4. If  $k = 0$  then return solution
5. Go to step 2

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## Deterministic Dynamic Programming – Basic Algorithm

### Graphical illustration of the solution procedure



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## Arc Cost Calculations

There are two ways for calculating the arc costs

- ▶ Calculate the exact control signal and cost for each arc.
  - Quasi-static approach
- ▶ Make a grid over the control signal and interpolate the cost for each arc.
  - Forward calculation approach

Matlab implementation – it is important to utilize matrix calculations

- ▶ Calculate the whole bundle of arcs in one step
- ▶ Add boundary and constraint checks

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## Pros

- ▶ Globally optimal, for all initial conditions
- ▶ Can handle nonlinearities and constraints
- ▶ Time complexity grows linearly with horizon
- ▶ Use output and solution as reference for comparison

## Cons

- ▶ Non causal
- ▶ Time complexity grows “exponentially” with number of states

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- ▶ Problem 200s with discretization  $\Delta t = 1s$ .
- ▶ Control signal discretized with 10 points.
- ▶ Statespace discretized with 1000 points.
- ▶ One evaluation of the model takes  $1\mu s$
- ▶ Solution time:
  - ▶ Brute force:  
Evaluate all possible combinations of control sequences.  
Number of evaluations,  $10^{200}$  gives  $\approx 3 \cdot 10^{186}$  years.
  - ▶ Dynamic programming:  
Number of evaluations:  $200 \cdot 10 \cdot 1000$  gives 2 s.

(Example contributed by ETH)

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## The Provided Tools for Hand-in Assignment 2

### Task:

Investigate optimal control of one parallel and one series hybrid configuration in different driving profiles.

- ▶ Some Matlab-functions provided
  - ▶ Skeleton file for defining the problems
  - ▶ 2 DDP solvers, 1-dim and 2-dim.
  - ▶ 2 skeleton files for calculating the arc costs for parallel and serial hybrids

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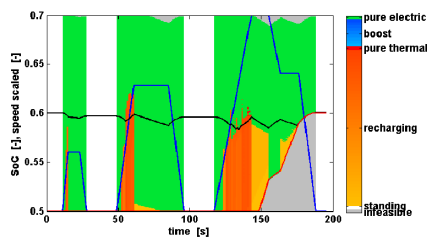
### Case Studies

- Energy Management of a Parallel Hybrid

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## Parallel Hybrid Example

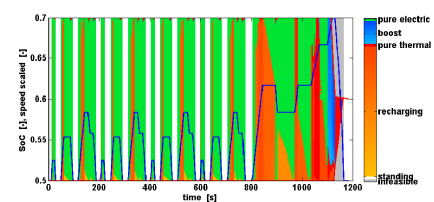
- ▶ Fuel-optimal torque split factor  $u(SOC, t) = \frac{T_{g-motor}}{T_{gearbox}}$
- ▶ ECE cycle
- ▶ Constraints  $SOC(t = t_f) \geq 0.6$ ,  $SOC \in [0.5, 0.7]$



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## Parallel Hybrid Example

- ▶ Fuel-optimal torque split factor  $u(SOC, t) = \frac{T_{g-motor}}{T_{gearbox}}$
- ▶ NEDC cycle
- ▶ Constraints  $SOC(t = t_f) = 0.6$ ,  $SOC \in [0.5, 0.7]$



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