

Vehicle Propulsion Systems

Lecture 6

Supervisory Control Algorithms

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Outline

Repetition

Supervisory Control Algorithms

Heuristic Control Approaches

Optimal Control Strategies

Analytical solutions to Optimal Control Problems

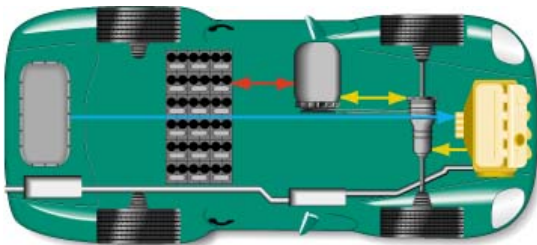
ECMS – Equivalent Consumption Minimization Strategy

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Hybrid Electrical Vehicles – Parallel

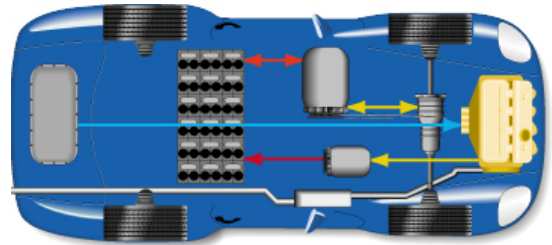
- ▶ Two parallel energy paths
- ▶ One state in QSS framework, state of charge



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Hybrid Electrical Vehicles – Serial

- ▶ Two paths working in parallel
- ▶ Decoupled through the battery
- ▶ Two states in QSS framework, state of charge & Engine speed

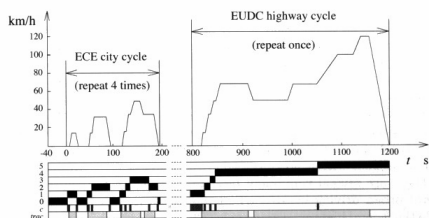


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Optimization, Optimal Control, Dynamic Programming

What gear ratios give the lowest fuel consumption for a given driving cycle?

– Problem presented in appendix 8.1



Problem characteristics

- ▶ Countable number of free variables, $i_{g,j}$, $j \in [1, 5]$
- ▶ A “computable” cost, $m_f(\dots)$
- ▶ A “computable” set of constraints, model and cycle
- ▶ The formulated problem

$$\begin{aligned} \min_{i_{g,j}, j \in [1,5]} \quad & m_f(i_{g,1}, i_{g,2}, i_{g,3}, i_{g,4}, i_{g,5}) \\ \text{s.t.} \quad & \text{model and cycle is fulfilled} \end{aligned}$$

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General problem formulation

- ▶ Performance index

$$J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t) dt$$

- ▶ System model (constraints)

$$\frac{d}{dt}x = f(x(t), u(t), t), x(t_a) = x_a$$

- ▶ State and control constraints

$$u(t) \in U(t)$$

$$x(t) \in X(t)$$

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Optimal Control – Problem Motivation

Car with gas pedal $u(t)$ as control input:

How to drive from A to B on a given time with minimum fuel consumption?

- ▶ Infinite dimensional decision variable $u(t)$.

- ▶ Cost function $\int_0^T \dot{m}_f(t) dt$

- ▶ Constraints:

- ▶ Model of the car (the vehicle motion equation)

$$\begin{aligned} m_v \frac{d}{dt}v(t) &= F_t(v(t), u(t)) - (F_a(v(t)) + F_r(v(t)) + F_g(x(t))) \\ \frac{d}{dt}x(t) &= v(t) \\ \dot{m}_f &= f(v(t), u(t)) \end{aligned}$$

- ▶ Starting point $x(0) = A$
- ▶ End point $x(t_r) = B$
- ▶ Speed limits $v(t) \leq g(x(t))$
- ▶ Limited control action $0 \leq u(t) \leq 1$

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Dynamic programming – Problem Formulation

- ▶ Optimal control problem

$$\min J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t) dt$$

$$\text{s.t. } \frac{d}{dt}x = f(x(t), u(t), t)$$

$$x(t_a) = x_a$$

$$u(t) \in U(t)$$

$$x(t) \in X(t)$$

- ▶ $x(t)$, $u(t)$ functions on $t \in [t_a, t_b]$

- ▶ Search an approximation to the solution by discretizing

- ▶ the state space $x(t)$
- ▶ and maybe the control signal $u(t)$

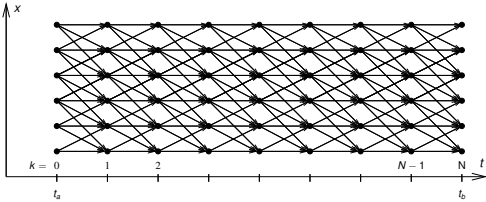
in both amplitude and time.

- ▶ The result is a combinatorial (network) problem

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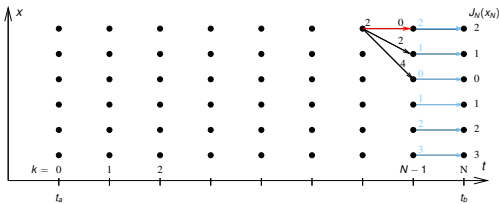
$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$
$$x_{k+1} = f_k(x_k, u_k)$$

Algorithm idea:
Start at the end and proceed backwards in time to evaluate the optimal cost-to-go and the corresponding control signal.



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Graphical illustration of the solution procedure



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Arc Cost Calculations

- There are two ways for calculating the arc costs
- Calculate the exact control signal and cost for each arc.
–Quasi-static approach
 - Make a grid over the control signal and interpolate the cost for each arc.
–Forward calculation approach

Matlab implementation – it is important to utilize matrix calculations

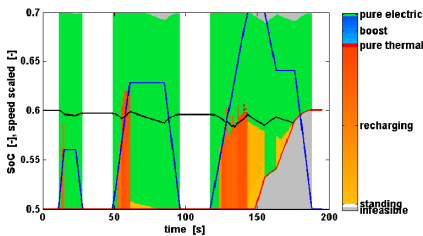
- Calculate the whole bundle of arcs in one step
- Add boundary and constraint checks

2D and 3D grid examples on whiteboard

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Parallel Hybrid Example

- Fuel-optimal torque split factor $u(SOC, t) = \frac{T_{e-motor}}{T_{gearbox}}$
- ECE cycle
- Constraints $SOC(t = t_f) \geq 0.6, SOC \in [0.5, 0.7]$



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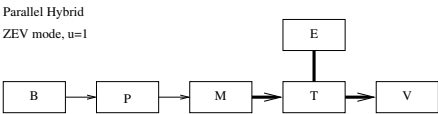
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Parallel Hybrid – Modes and Power Flows

The different modes for a parallel hybrid

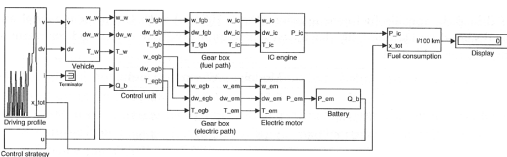
$$u \approx P_{batt} / P_{vehicle}$$

Battery drive mode (ZEV)



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Control algorithms



- Determining the power split ratio u

$$u_j(t) = \frac{P_j(t)}{P_{m+1}(t) + P_i(t)} \quad (4.110)$$

- Clutch engagement disengagement $B_c \in \{0, 1\}$
- Engine engagement disengagement $B_e \in \{0, 1\}$

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Strategies for the Parallel Hybrid

Power split u , Clutch B_c , Engine B_e

Mode	u	B_e	B_c
1 ICE	0	1	1
2a ZEV	1	0	0
2b ZEV	1	0	1
3 Power assist	$[0, 1]$	1	1
4 Recharge	< 0	1	1
5a Regenerative braking	1	0	0
5a Regenerative braking	1	0	1

All practical control strategies have engine shut off when the torque at the wheels are negative or zero; standstill, coasting and braking.

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- ▶ Non-causal controllers
 - ▶ Detailed knowledge about future driving conditions.
 - ▶ Position, speed, altitude, traffic situation.
 - ▶ Uses:
 - Regulatory drive cycles, public transportation, long haul operation, GPS based route planning.
- ▶ Causal controllers
 - ▶ No knowledge about the future...
 - ▶ Use information about the current state.
 - ▶ Uses:
 - "The normal controller", on-line, in vehicles without planning

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- ▶ Heuristic controllers
 - Causal
 - State of the art in most prototypes and mass-production
- ▶ Optimal controllers
 - Often non-causal
 - Solutions exist for simplifications
- ▶ Sub-optimal controllers
 - Often causal

On-going work to include optimal controllers in prototypes

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Some Comments About the Problem

- ▶ Difficult problem
- ▶ Unsolved problem for causal controllers
- ▶ Rich body of engineering reports and research papers on the subject
 - This can clearly be seen when reading chapter 7!

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Heuristic Control Approaches

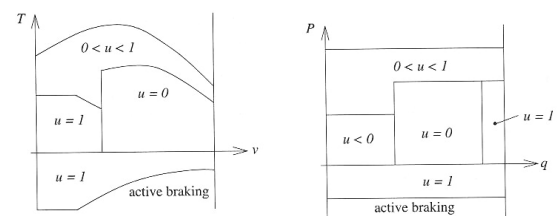
Operation usually depends on a few vehicle operation

- ▶ Rule based:
 - Nested if-then-else clauses
 - if $v < v_{low}$ then use electric motor ($u=1$).
 - else...
- ▶ Fuzzy logic based
 - Classification of the operating condition into fuzzy sets.
 - Rules for control output in each mode.
 - Defuzzification gives the control output.

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Heuristic Control Approaches

- ▶ Parallel hybrid vehicle (electric assist)



- ▶ Determine control output as function of some selected state variables:
 - vehicle speed, engine speed, state of charge, power demand, motor speed, temperature, vehicle acceleration, torque demand.

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Heuristic Control Approaches – Concluding Remarks

- ▶ Easy to conceive
- ▶ Relatively easy to implement
- ▶ Result depends on the thresholds
- ▶ Proper tuning can give good fuel consumption reduction and charge sustainability
- ▶ Performance varies with cycle and driving condition
 - Not robust
- ▶ Time consuming to develop and tune for advanced hybrid configurations

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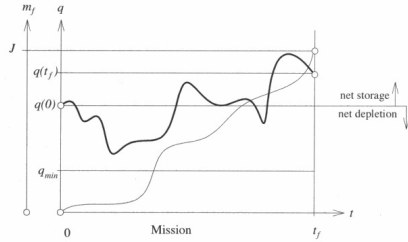
ECMS – Equivalent Consumption Minimization Strategy

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Consider a driving mission

► Variables.

Control signal – $u(t)$, System state – $x(t)$, State of charge – $q(t)$ (is a state).



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Formulating the Optimal Control Problem

–What is the optimal behaviour? Defines *Performance index J*.

► Minimize the fuel consumption

$$J = \int_0^{t_f} \dot{m}_f(t, u(t)) dt$$

► Balance between fuel consumption and emissions

$$J = \int_0^{t_f} \left[\dot{m}_f(t, u(t)) + \alpha_{CO} \dot{m}_{CO}(x(t), u(t)) + \alpha_{NO} \dot{m}_{NO}(x(t), u(t)) + \alpha_{HC} \dot{m}_{HC}(x(t), u(t)) \right] dt$$

► Include driveability criterion

$$J = \int_0^{t_f} \dot{m}_f(t, u(t)) + \beta \left(\frac{d}{dt} a(t) \right)^2 dt$$

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First Solution to the Problem

► Minimize the fuel consumption

$$J = \int_0^{t_f} \dot{m}_f(t, u(t)) dt$$

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Including constraints

► Hard or soft constraints

$$\min J(u) = \int_0^{t_f} L(t, u(t)) dt$$

s.t. $q(0) = q(t_f)$

$$\min J(u) = \phi(q(t_f)) + \int_0^{t_f} L(t, u(t)) dt$$

► How to select $\phi(q(t_f))$?

$$\phi(q(t_f)) = \alpha (q(t_f) - q(0))^2$$

penalizes high deviations more than small, independent of sign

$$\phi(q(t_f)) = w (q(0) - q(t_f))$$

penalizes battery usage, favoring energy storage for future use

► One more feature from the last one

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Including constraints

► Including battery penalty according to

$$\phi(q(t_f)) = w (q(0) - q(t_f)) = -w \int_0^{t_f} \dot{q}(t) dt$$

enables us to rewrite

$$\min J(u) = \int_0^{t_f} L(t, u(t)) - w \dot{q}(t) dt$$

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Constraints That are Also Included

► State equation $\dot{x} = f(x)$ is also included – From Lecture 5

► Consider hybrid with only one state, SoC

$$\min J(u) = \phi(q(t_f), t_f) + \int_0^{t_f} L(t, u(t)) dt$$

$$\begin{aligned} \text{s.t. } & \frac{d}{dt} q = f(t, q(t), u(t)) \\ & u(t) \in U(t) \\ & q(t) \in Q(t) \end{aligned}$$

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Analytical Solutions to Optimal Control Problems

► Core of the problem

$$\begin{aligned} \min J(u) &= \phi(q(t_f), t_f) + \int_0^{t_f} L(t, u(t)) dt \\ \text{s.t. } \dot{q}(t) &= f(t, q(t), u(t)) \end{aligned}$$

► Hamiltonian from optimal control theory

$$H(t, q(t), u(t), \mu(t)) = L(t, u(t)) + \mu(t) f(t, q(t), u(t))$$

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Analytical Solutions to Optimal Control Problems

- Hamiltonian

$$H(t, q(t), u(t), \mu(t)) = L(t, u(t)) + \mu(t) f(t, q(t), u(t))$$

- Solution (theory from Appendix B)

$$u(t) = \arg \min_u H(t, q(t), u(t), \mu(t))$$

with

$$\dot{\mu}(t) = -\frac{\partial}{\partial q} f(t, q(t), u(t))$$

$$\dot{q}(t) = f(t, q(t), u(t))$$

- If $\frac{\partial}{\partial q} f(t, q(t), u(t)) = 0$ the problem becomes simpler
 μ becomes a constant μ_0 , search for it when solving

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Analytical Solutions to Optimal Control Problems

- μ_0 depends on the (soft) constraint

$$\mu_0 = \frac{\partial}{\partial q(t_f)} \phi(q(t_f)) = \text{/special case/} = -w$$

- Different efficiencies

$$\mu_0 = \frac{\partial}{\partial q(t_f)} \phi(q(t_f)) = \begin{cases} -w_{dis}, & q(t_f) > q(0) \\ -w_{chg}, & q(t_f) < q(0) \end{cases}$$

- Introduce equivalence factor (scaling) by studying battery and fuel power

$$s(t) = -\mu(t) \frac{H_{LHV}}{V_b Q_{max}}$$

ECMS – Equivalent Consumption Minimization Strategy

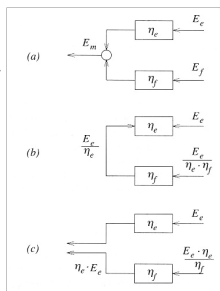
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Determining Equivalence Factors I

Constant engine and battery efficiencies

$$s_{dis} = \frac{1}{\eta_e \eta_f}$$

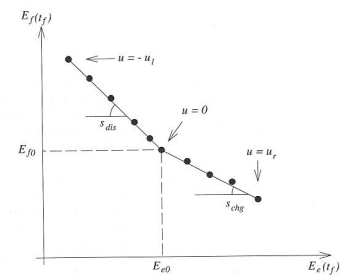
$$s_{chg} = \frac{\eta_e}{\eta_f}$$



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Determining Equivalence Factors II

- Collecting battery and fuel energy data from test runs with constant u gives a graph

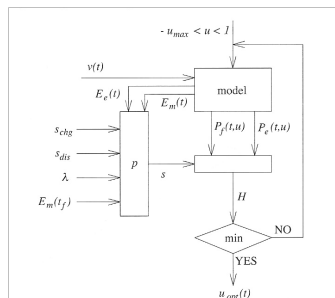


- Slopes determine s_{dis} and s_{chg} .

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ECMS On-line Implementation

Flowchart



There is also a T-ECMS (telemetry-ECMS)

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