Graph Theoretical Methods for Finding Analytical Redundancy Relations in Overdetermined Differential Algebraic Systems

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Abstract - One approach for design of diagnosis systems is to use residuals based on analytical redundancy. Overdetermined systems of equations provide analytical redundancy and by using minimal overdetermined subsystems, sensitivity to few faults is obtained. In this paper, overdetermined differential algebraic systems are considered and their structure is represented by bipartite graphs with equations and unknowns as node sets. By differentiating equations, a new set is formed, that is an overdetermined static algebraic system if derivatives of unknown signals are considered as separate independent variables. The task to derive analytical redundancy relations is thereby reduced to an algebraic problem. It is desirable to differentiate the equations as few times as possible and it is shown that there exists a unique minimally differentiated overdetermined system.

Keywords— **Structural analysis, consistency based diagnosis, differential algebraic equations, bipartite graphs.**

I. Introduction

In model based diagnosis, a model of the fault free system is compared to observations [BLA 03], [GER 98]. If the observations and the model are inconsistent, then it is concluded that a fault is present. To get inconsistency, redundancy is needed. Consider for example the algebraic system

$$\begin{aligned} x_1 &= x_2^2 + u \\ x_2 &= e^{x_1} \\ y &= x_1 \end{aligned} \qquad (1)$$

where u and y are known, and x_1 and x_2 are unknown variables. This set contains more equations than unknowns and is overdetermined. The redundancy in the equations can be used to check if u and y are consistent with the model. If the unknowns, x_1 and x_2, are eliminated in the equation system (1), then the equation

$$y - e^{2\,y} - u = 0 \qquad (2)$$

is obtained. If (2) is not fulfilled, then u and y are not consistent with the model (1). The equation (2) is an example of an analytical redundancy relation, also called parity relation or consistency relation in literature.

We have seen how consistency can be checked in an algebraic system. Now we will show how to treat differential algebraic systems and the next example illustrates the basic ideas of the approach presented in this paper. The differential algebraic system

equation E	unknown X: x	y	λ
e_1	X		X
e_2		X	X
e_3	X	X	
e_4	X	X	

Fig. 1. The structural model for the pendulum.

$$\begin{aligned} e_1: &\quad L^{-1}\lambda(t)\,x(t) + m\,x^{(2)}(t) &= 0 \\ e_2: &\quad L^{-1}\lambda(t)\,y(t) + m\,y^{(2)}(t) + g\,m &= 0 \\ e_3: &\quad x(t)^2 + y(t)^2 - L^2 &= 0 \\ e_4: &\quad L^{-2}(x(t)\,y^{(1)}(t) - y(t)\,x^{(1)}(t)) - z(t) &= 0 \end{aligned} \qquad (3)$$

models the motion of a pendulum and a angular velocity measurement z. Here $x(t)$, $y(t)$, and $\lambda(t)$ are the unknown state variables, L is the length, m is the mass, and g is the gravitational constant. The set of equations $\{e_1, e_2, e_3, e_4\}$ will be denoted E and the set of unknown states $\{x(t), y(t), \lambda(t)\}$ by X.

To present the structure of the model, a bipartite graph is used with equations and unknowns as node sets [CAS 97], [KRY 03]. There is an edge between an equation and an unknown if the unknown is contained in the equation. Figure 1 shows the graph, for the system (3), represented as an incidence matrix where X marks an edge.

Now some important structural properties will be defined. If E is a set of equations and X is a set of variables then

$$\text{var}_X(E) := \{x \in X | x \text{ is included in an } e \in E\} \qquad (4)$$

In consistency based diagnosis, redundancy in the model is used and this motivates the following definition.

Definition 1 (Structurally Overdetermined) A finite set of equations E is *structurally overdetermined* (SO) with respect to the set of variables X if $|E| > |\text{var}_X(E)|$.

By considering small SO sets, consistency tests will be sensitive to few faults. This is desirable when identifying which fault that has occurred.

Definition 2 (Minimal Structurally Overdetermined) A set of equations E is a *minimal structurally overdetermined*

(MSO) set with respect to X if E is structurally overdetermined with respect to X and no proper subset of E is structurally overdetermined with respect to X.

Equation system (3), with the structure as shown in Figure 1, is an example of an MSO set with respect to $\{x(t), y(t), \lambda(t)\}$ and there is redundancy in the model. However, the algebraic elimination procedure, used in the first example, can not be used immediately because of the presence of differentiated states.

By considering states and their derivatives as separate independent variables, analytical redundancy relations can be derived by using algebraic elimination if the set is SO [KRY 02]. The incidence matrix of the bipartite graph for system (3), using this approach, is shown in Figure 2.

equation	unknown						
	x	$x^{(1)}$	$x^{(2)}$	y	$y^{(1)}$	$y^{(2)}$	λ
e_1	X		X				X
e_2				X		X	X
e_3	X			X			
e_4	X	X		X	X		

Fig. 2. The structural model where the states and their derivatives are distinguished.

This algebraic system is not SO, but by differentiating equations with respect to t, new equations are obtained, for example

$$e_3^{(1)} : 2xx^{(1)} + 2yy^{(1)} = 0$$

By starting with an MSO set E w.r.t. X and differentiating all equations with respect to t multiple times, the set of differentiated equations will eventually grow faster than the set of differentiated states. Therefore, it is always possible to obtain an SO set in this way and this set contains an MSO subset.

An elementary algorithm to find an MSO set is to differentiate all equations until there exists a subset that is SO. The Dulmage-Mendelsohn decomposition can be used to determine if there exists a subset that is SO [DUL 58]. It follows from the results in this paper that this set is also an MSO set and that it is minimally differentiated as described in the next section.

It is easy to obtain an upper limit of the number of differentiations that are needed to obtain an MSO set. For the example, it can be noticed that after differentiating all four equations four times, 20 equations are obtained with 19 unknowns. Hence, this set contains an MSO set. However, Figure 3 shows an MSO set where the order of all the derivatives are at most three.

In this particular case, the equations in the MSO set are all polynomials in the unknowns and the unknowns can therefore be eliminated using for example Gröbner basis [COX 97]. A consistency relation derived in this way is

$$m\, z(t)^2 (g^2 - L^2\, (z^{(1)}(t))^2) - L^2\, m\, (z^{(2)}(t))^2 = 0$$

It is should be pointed out that the structural analysis outlined above is not restricted to polynomials and can be applied to general non-linear problems.

From now on, the original set E of equations is assumed to be an MSO set with respect to X, as shown in Figure 1. We assume that the equations E have been differentiated and an MSO set E_d with respect to X_d has been found as in Figure 3. The set E_d is partitioned into two sets E_d^l and E_d^m, where E_d^m contains the highest derivative of each equation in E. The set X_d is partitioned into X_d^l and X_d^m in a similar way.

II. Uniqueness of Differentiated MSO Sets

In the previous section, it was shown how to obtain an MSO set E_d w.r.t X_d, where different derivatives of equations and states are distinguished. In the following section some aspects of uniqueness are investigated.

In general, the bipartite graph can be partitioned as in Figure 4, using the notation introduced at the end of the previous section.

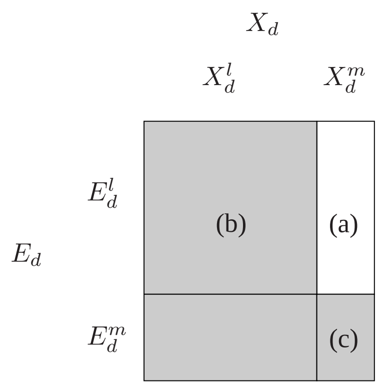

Fig. 4. Partition of the graph.

The structure of the sub-graphs (a), (b), and (c) are revealed in a sequence of lemmas, which leads to the main result formulated in Theorem 1. There, it is shown that there exists a unique minimally differentiated MSO set. The set is minimally differentiated in the following sense. For any other MSO set, derived from the same original set of equations, the order of the highest derivative of each equation is strictly greater than the order of the derivatives of the same equation in the minimally differentiated set. The MSO set shown in Figure 3 is a minimally differentiated MSO set.

The reason for studying minimally differentiated sets is that some of the equations contain measured signals, for which the derivatives are difficult to estimate in a noisy environment. It is therefore natural to consider the problem of minimizing the derivatives of a subset of equations that contains measured signals. However, it follows from what was

equation		unknown X_d									
		X_d^l							X_d^m		
	E_d	x	$x^{(1)}$	$x^{(2)}$	y	$y^{(1)}$	$y^{(2)}$	λ	$x^{(3)}$	$y^{(3)}$	$\lambda^{(1)}$
E_d^l	e_1	X		X				X			
	e_2				X		X	X			
	e_3	X			X						
	$e_3^{(1)}$	X	X		X	X					
	$e_3^{(2)}$	X	X	X	X	X	X				
	e_4	X	X		X	X					
	$e_4^{(1)}$	X	X	X	X	X	X				
E_d^m	$e_1^{(1)}$	X	X					X	X		X
	$e_2^{(1)}$				X	X		X		X	X
	$e_3^{(3)}$	X	X	X	X	X	X		X	X	
	$e_4^{(2)}$	X	X	X	X	X	X		X	X	

Fig. 3. An MSO set in a differentiated structural model.

said above, that the solution to this problem is the same as for the original problem.

Now, the sequence of lemmas, mentioned above, will be presented. The first result is that there is only one redundant equation in an MSO set.

Lemma 1: If E is an MSO set w.r.t. X, then $|E| = |\text{var}_X(E)| + 1$.

Proof: Since E is SO w.r.t. X, it follows that

$$|E| \geq |\text{var}_X(E)| + 1$$

If equality holds, then there is nothing to prove. Assume that E is MSO w.r.t. X and that

$$|E| > |\text{var}_X(E)| + 1$$

Take any $E' \subset E$ such that

$$|E'| = |\text{var}_X(E)| + 1$$

Since $E' \subset E$, it follows that $|\text{var}_X(E')| \leq |\text{var}_X(E)|$ which implies that

$$|E'| = |\text{var}_X(E)| + 1 \geq |\text{var}_X(E')| + 1$$

This means that E' is SO which contradicts the assumption and the lemma follows. ■

The next Lemma shows that the sub-graph (a) in Figure 4 has no edges.

Lemma 2: $\text{var}_{X_d^m}(E_d^l) = \varnothing$

Proof: Assume that

$$x_i^{(l)} \in \text{var}_{X_d^m}(E_d^l) \subset X_d^m$$

Then $x_i^{(l)} \in \text{var}_{X_d^m}(e_j^{(k)})$ for some $e_j^{(k)} \in E_d^l$. Since $e_j^{(k)} \in E_d^l$, it follows that $e_j^{(k+p)} \in E_d^m$ for some $p \in \mathbb{Z}_+$. This implies that $x_i^{(l+p)} \in \text{var}_{X_d}(E_d)$. But this contradicts the assumption that $x_i^{(l)} \in X_d^m$, which completes the proof. ■

Now we show that the two node sets in the sub-graph (c) are of the same size as in the original graph. It is also shown that the degree of the variable nodes are nonzero.

Lemma 3: $|E_d^m| = |\text{var}_{X_d^m}(E_d^m)| + 1$ and $\text{var}_{X_d^m}(E_d^m) = X_d^m$.

Proof: From the definition of X_d^m it follows that

$$|X| = |X_d^m| \quad (5)$$

and $X_d^m = \text{var}_{X_d^m}(E_d)$. This,

$$\text{var}_{X_d^m}(E_d) = \text{var}_{X_d^m}(E_d^l) \cup \text{var}_{X_d^m}(E_d^m)$$

and Lemma 2 imply

$$X_d^m = \text{var}_{X_d^m}(E_d^m) \quad (6)$$

which is the second conclusion of this lemma.

From the definition of E_d^m, it follows that

$$|E_d^m| = |E| \quad (7)$$

The definition of X, (5), and (6) imply

$$|\text{var}_X(E)| = |\text{var}_{X_d^m}(E_d^m)| \quad (8)$$

Since E is MSO w.r.t. X, Lemma 1 implies

$$|E| = |\text{var}_X(E)| + 1 \quad (9)$$

Now, eliminating $|E|$ and $|\text{var}_X(E)|$ by using (7) and (8),

$$|E_d^m| = |\text{var}_{X_d^m}(E_d^m)| + 1$$

is obtained and the Lemma follows. ■

The next lemma states that the cardinality of the two node sets in the sub-graph (b) are the same and that the degree of the variable nodes are nonzero. That the degrees of the equation nodes are nonzero follows trivially from Lemma 2 and the fact that each equation has to contain at least one unknown.

Lemma 4: $|E_d^l| = |\mathrm{var}_{X_d}(E_d^l)|$ and $\mathrm{var}_{X_d}(E_d^l) = X_d^l$.

Proof: Lemma 1 applied to the MSO set E_d implies that

$$|E_d^m| + |E_d^l| = |X_d^m| + |X_d^l| + 1$$

and Lemma 3 implies that

$$|E_d^m| = |X_d^m| + 1$$

From these two equalities, it follows that

$$|E_d^l| = |X_d^l| \tag{10}$$

Since E_d is an MSO set with respect to X_d, it follows that $E_d^l \subsetneq E_d$ is not SO with respect to X_d, i.e.

$$|E_d^l| \leq |\mathrm{var}_{X_d}(E_d^l)| \tag{11}$$

Lemma 2 implies that

$$\mathrm{var}_{X_d}(E_d^l) = \mathrm{var}_{X_d^l}(E_d^l) \subset X_d^l$$

By using this in (11), it follows that

$$|E_d^l| \leq |\mathrm{var}_{X_d}(E_d^l)| \leq |X_d^l| \tag{12}$$

This and (10) imply that

$$|E_d^l| = |\mathrm{var}_{X_d}(E_d^l)| = |X_d^l|$$

Finally, since $\mathrm{var}_{X_d}(E_d^l) \subset X_d^l$ and $|\mathrm{var}_{X_d}(E_d^l)| = |X_d^l|$ the lemma follows.

■

In the example, the sub-graph (c) is isomorphic to the original graph in Figure 1. In general, the edges of (c) is a subset of the set of edges corresponding to the original graph. However, the following result shows that (c) still represents an MSO set.

Lemma 5: The set E_d^m is an MSO set w.r.t. X_d^m.

Proof: Assume that E_{d1}^m is SO w.r.t. X_d^m and $E_{d1}^m \subsetneq E_d^m$. The idea is to show that these assumptions imply that $E_{d1}^m \cup E_d^l$ is SO w.r.t. X_d which contradicts that E_d is MSO w.r.t. X_d. The assumption that E_{d1}^m is SO w.r.t. X_d^m and Lemma 4 imply that

$$\begin{aligned} |E_{d1}^m \cup E_d^l| &< |\mathrm{var}_{X_d^m}(E_{d1}^m)| + |\mathrm{var}_{X_d^l}(E_d^l)| \\ &= |\mathrm{var}_{X_d^m}(E_{d1}^m) \cup \mathrm{var}_{X_d^l}(E_d^l)| \end{aligned} \tag{13}$$

From Lemma 4, it follows that

$$\mathrm{var}_{X_d^l}(E_{d1}^m) \subset \mathrm{var}_{X_d^l}(E_d^l) = X_d^l$$

From this, Lemma 2, and that

$$\mathrm{var}_{X_d^l}(E_{di}) \cup \mathrm{var}_{X_d^m}(E_{di}) = \mathrm{var}_{X_d}(E_{di})$$

for any E_{di}, it follows that

$$\mathrm{var}_{X_d^m}(E_{d1}^m) \cup \mathrm{var}_{X_d^l}(E_d^l) = \mathrm{var}_{X_d}(E_{d1}^m \cup E_d^l)$$

If the left-hand side of this expression is substituted into (13), then it follows that $E_{d1}^m \cup E_d^l$ is SO w.r.t. X_d which contradicts that E_d is an MSO set w.r.t. X_d. Hence the lemma follows.

■

Consider two different MSO sets derived from the same equations. It follows from the next result that the two subgraphs (c) in Figure 4 corresponding to the two MSO sets are isomorphic.

Lemma 6: There exist integers $\alpha_1, \ldots, \alpha_n$ such that for any MSO set E_d derived from E, the set E_d^m admits the representation

$$E_d^m = \{e_1^{(\alpha_1+k)}, \ldots, e_n^{(\alpha_n+k)}\}$$

for some integer k.

Proof: Let E_{d1} and E_{d2} be two arbitrary MSO sets with the corresponding subsets

$$E_{d1}^m = \{e_1^{(\alpha_1)}, \ldots, e_n^{(\alpha_n)}\}$$

and

$$E_{d2}^m = \{e_1^{(\beta_1)}, \ldots, e_n^{(\beta_n)}\}$$

To prove the lemma, it is sufficient to show that $\beta_i - \alpha_i = k$ for some k. Let $k = \max_i(\beta_i - \alpha_i)$. Either $\alpha_i = \beta_i$ for all i and there is nothing to prove, or the MSO sets can be enumerated so that $k > 0$. We can therefore assume that $k > 0$. Let

$$E_0 = \{e_i^{(\alpha_i)} : \beta_i - \alpha_i = k\}$$

and

$$X_0 = \mathrm{var}_{X_{d1}^m}(E_0)$$

It holds that

$$E_0^{(k)} := \{e_i^{(\alpha_i+k)} : \beta_i - \alpha_i = k\} = \{e_i^{(\beta_i)} : \beta_i - \alpha_i = k\}$$

and consequently it follows that

$$E_0^{(k)} \subset E_{d2}^m \tag{14}$$

Recall that $\beta_i = \alpha_i + k$ for $e^{(\beta_i)} \in E_0^{(k)}$ and that

$\beta_i < \alpha_i + k$ for $e^{(\beta_i)} \in E_{d2}^m \backslash E_0^{(k)}$. Assume that

$$x_i^{(\gamma)} \in X_0 = \mathrm{var}_{X_{d1}^m}(E_0)$$

It follows that $x_i^{(\gamma+k)} \in X_{d2}^m$ and hence

$$X_0^{(k)} \subset X_{d2}^m \tag{15}$$

It follows also that $x_i^{(\gamma+k)} \notin \text{var}_{X_{d2}^m}(E_{d2}^m \setminus E_0^{(k)})$ and consequently it holds that

$$\text{var}_{X_0^{(k)}}(E_{d2}^m \setminus E_0^{(k)}) = \emptyset \tag{16}$$

Assume now that $\alpha_i - \beta_i = k$ does not hold for all i or equivalently

$$E_0 \neq E_{d1}^m \tag{17}$$

We will show that this contradicts (16). The set E_{d1}^m is an MSO set w.r.t. X_{d1}^m according to Lemma 5. Together with assumption (17) this implies that $|E_0| \leq |X_0|$ and

$$|E_0^{(k)}| \leq |X_0^{(k)}|$$

Moreover E_{d2}^m is an MSO set w.r.t. X_{d2}^m according to Lemma 5 and hence

$$|E_{d2}^m| > |X_{d2}^m|$$

It follows from the two inequalities above and the set relations (14) and (15) that

$$|E_{d2}^m \setminus E_0^{(k)}| = |E_{d2}^m| - |E_0^{(k)}| > |X_{d2}^m| - |X_0^{(k)}| = |X_{d2}^m \setminus X_0^{(k)}|$$

This implies that

$$|E_{d2}^m \setminus E_0^{(k)}| > |\text{var}_{X_{d2}^m \setminus X_0^{(k)}}(E_{d2}^m \setminus E_0^{(k)})|$$

and since $E_{d2}^m \setminus E_0^{(k)}$ is not SO we have

$$|E_{d2}^m \setminus E_0^{(k)}| \leq |\text{var}_{X_{d2}^m}(E_{d2}^m \setminus E_0^{(k)})|$$

It follows from these two inequalities that

$$\begin{aligned} |\text{var}_{X_0^{(k)}}(E_{d2}^m \setminus E_0^{(k)})| =& |\text{var}_{X_{d2}^m}(E_{d2}^m \setminus E_0^{(k)})| \\ & - |\text{var}_{X_{d2}^m \setminus X_0^{(k)}}(E_{d2}^m \setminus E_0^{(k)})| \\ >& |E_{d2}^m \setminus E_0^{(k)}| - |E_{d2}^m \setminus E_0^{(k)}| = 0 \end{aligned}$$

This contradicts (16) and the proof is complete. ■

Now we can prove the main result of this paper, outlined in the introduction of this section.

Theorem 1: Given an MSO set E w.r.t. X, there exists a unique minimally differentiated MSO set E_d w.r.t. X_d.

Proof: Assume that E_{d1} and E_{d2} are two minimally differentiated MSO sets. According to Lemma 6 the corresponding sets E_{d1}^m and E_{d2}^m coincide and the notation E_d^m is used for both. Let X_d' be defined as $X_{d1} \cup X_{d2}$. The set $E_{d1}^l \cup E_{d2}^l$ is not SO, since this would imply that there exists a subset of $E_{d1}^l \cup E_{d2}^l$ that is an MSO set, which contradicts that E_{d1} and E_{d2} are both minimally differentiated. Hence

$$|E_{d1}^l \cup E_{d2}^l| \leq |\text{var}_{X_d'}(E_{d1}^l \cup E_{d2}^l)|$$

Using this inequality, Lemma 4, and that

$$\text{var}_{X_d'}(E_{d1}^l \cup E_{d2}^l) = \text{var}_{X_d'}(E_{d1}^l) \cup \text{var}_{X_d'}(E_{d2}^l)$$

we get

$$\begin{aligned} |E_{d1}^l \cap E_{d2}^l| &= |E_{d1}^l| + |E_{d2}^l| - |E_{d1}^l \cup E_{d2}^l| \\ &\geq |\text{var}_{X_d'}(E_{d1}^l)| + |\text{var}_{X_d'}(E_{d2}^l)| \\ &\quad - |\text{var}_{X_d'}(E_{d1}^l) \cup \text{var}_{X_d'}(E_{d2}^l)| \\ &= |\text{var}_{X_d'}(E_{d1}^l) \cap \text{var}_{X_d'}(E_{d2}^l)| \end{aligned} \tag{18}$$

The set relation

$$\text{var}_{X_d'}(E_{d1} \cap E_{d2}) \subset \text{var}_{X_d'}(E_{d1}) \cap \text{var}_{X_d'}(E_{d2})$$

holds and it follows from Lemma 4 that

$$\begin{aligned} &\text{var}_{X_d'}(E_{d1}) \cap \text{var}_{X_d'}(E_{d2}) \\ &\quad = (\text{var}_{X_d'}(E_{d1}^l) \cap \text{var}_{X_d'}(E_{d2}^l)) \cup \text{var}_{X_d^m}(E_d^m) \end{aligned}$$

where $\text{var}_{X_d'}(E_{d1}^l) \cap \text{var}_{X_d'}(E_{d2}^l)$ and $\text{var}_{X_d^m}(E_d^m)$ are disjoint according to Lemma 2. This gives that

$$\begin{aligned} &|\text{var}_{X_d'}(E_{d1} \cap E_{d2})| \leq |\text{var}_{X_d'}(E_{d1}) \cap \text{var}_{X_d'}(E_{d2})| \\ &\quad = |\text{var}_{X_d'}(E_{d1}^l) \cap \text{var}_{X_d'}(E_{d2}^l)| + |\text{var}_{X_d^m}(E_d^m)| \end{aligned}$$

where

$$|\text{var}_{X_d'}(E_{d1}^l) \cap \text{var}_{X_d'}(E_{d2}^l)| \leq |E_{d1}^l \cap E_{d2}^l|$$

according to (18) and

$$|\text{var}_{X_d^m}(E_d^m)| < |E_d^m|$$

according to Lemma 3. It follows that

$$\begin{aligned} |\text{var}_{X_d'}(E_{d1} \cap E_{d2})| &< |E_{d1}^l \cap E_{d2}^l| + |E_d^m| \\ &= |(E_{d1}^l \cap E_{d2}^l) \cup E_d^m| \\ &= |E_{d1} \cap E_{d2}| \end{aligned}$$

Hence, $E_{d1} \cap E_{d2}$ is an SO set and can not be a proper subset of the MSO sets E_{d1} and E_{d2}. It follows that

$$E_{d1} = E_{d2} = E_{d1} \cap E_{d2}$$

and the proof is complete. ■

III. CONCLUSION

One approach for design of diagnosis systems is to use residuals based on analytical redundancy. Overdetermined systems of equations provide analytical redundancy and by using minimal overdetermined subsystems, sensitivity to few faults is obtained.

A method has been presented that reduces the problem of checking consistency of an overdetermined differential algebraic system into an algebraic problem. This is done by considering the unknowns and their derivatives as separate independent variables and differentiating equations in order to obtain an overdetermined system.

To present the structure of the algebraic system, a bipartite graph is used and properties of the graph have been investigated in the sequence of six lemmas. It is desirable to differentiate the equations as few times as possible, to avoid higher derivatives of measured signals. The main result is stated in Theorem 1, where it is shown that there exists a unique minimally differentiated MSO set. These MSO sets can be used to derive consistency relations, by using algebraic elimination methods.

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