Vehicle Propulsion Systems
Lecture 6
Deterministic Dynamic Programming and Some Examples

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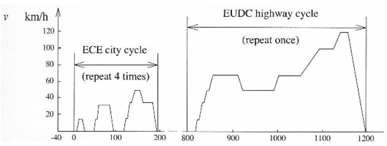
Outline

- Repetition
- “Traditional” Optimization
  - Problem motivation
  - Different Classes of Problems
- Optimal Control
  - Problem Motivation
- Deterministic Dynamic Programming
  - Problem setup and basic solution idea
  - Cost Calculation – Two Implementation Alternatives
  - The Provided Tools
- Case Studies
  - Energy Management of a Parallel Hybrid

Energy consumption for cycles



Numerical values for MVEG-95, ECE, EUDC

$$\text{air drag} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{v}_i^3\, h = \quad \{319, 82.9, 455\}$$

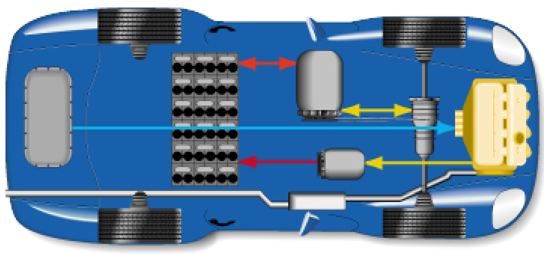
$$\text{rolling resistance} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{v}_i\, h = \quad \{.856, 0.81, 0.88\}$$

$$\text{kinetic energy} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{a}_i\, \bar{v}_i\, h = \quad \{0.101, 0.126, 0.086\}$$

$$\bar{E}_{MVEG\text{-}95} \approx A_f\, c_d\, 1.9 \cdot 10^4 + m_v\, c_r\, 8.4 \cdot 10^2 + m_v\, 10 \qquad kJ/100km$$

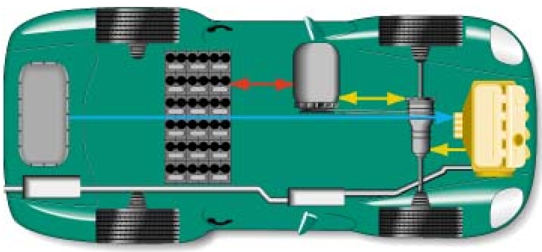
Hybrid Electrical Vehicles – Serial

- Two paths working in parallel
- Decoupled through the battery



Hybrid Electrical Vehicles – Parallel

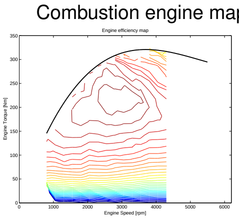
- Two parallel energy paths



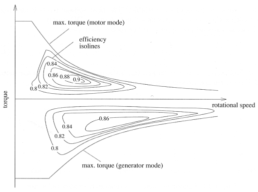
Component modeling

- Model energy (power) transfer and losses
- Using maps $\eta = f(T, \omega)$

Combustion engine map



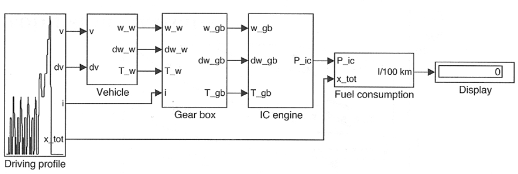
Electric motor map



- Using parameterized (scalable) models
  –Willans approach

Model implemented in QSS

Conventional powertrain



Efficient computations are important
–For example if we want to do optimization and sensitivity studies.

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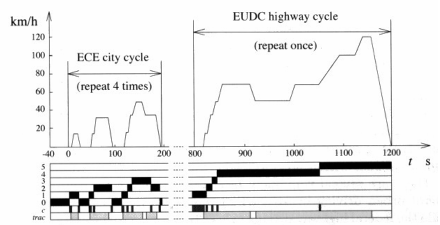
Problem motivation

What gear ratios give the lowest fuel consumption for a given drivingcycle?
–Problem presented in appendix 8.1



Problem characteristics

- Countable number of free variables, $i_{g,j}$, $j \in [1,5]$
- A "computable" cost, $m_f(\cdots)$
- A "computable" set of constraints, model and cycle
- The formulated problem

$$\begin{array}{ll} \min\limits_{i_{g,j},\, j\in[1,5]} & m_f(i_{g,1}, i_{g,2}, i_{g,3}, i_{g,4}, i_{g,5}) \\ \text{s.t.} & \text{model and cycle is fulfilled} \end{array}$$

Optimization – Linear Programming

- Linear problem

$$\begin{array}{llcl} \min\limits_{x} & c^T x & & \\ \text{s.t.} & A\,x & = & b \\ & x & \geq & 0 \end{array}$$

- Convex problem
- Much analyzed: existence, uniqueness, sensitivity
- Many algorithms: Simplex the most famous

- About the word *Programming*
  –The solution to a problem was called a program

Optimization – Non-Linear Programming

- Non-linear problem

$$\begin{array}{llcl} \min\limits_{x} & f(x) & & \\ \text{s.t.} & g(x) & = & 0 \\ & x & \geq & 0 \end{array}$$

- For convex problems
  –Much analyzed: existence, uniqueness, sensitivity
  –Many algorithms
- For non-convex problems
  –Some special problems have solutions
  –Local optimum is not necessarily a global optimum

Mixed Integer and Combinatorial Optimziation

- Problem

$$\begin{array}{llcl} \min\limits_{x} & f(x,y) & & \\ \text{s.t.} & g(x,y) & = & 0 \\ & x & \geq & 0 \\ & y & \in & Z^+ \end{array}$$

- Inherently non-convex y
  Generally hard problems to solve.
- Much analyzed
  –Existence, uniqueness, sensitivity
  –Many types of problems
  –Many different algorithms

Some comments on problem solver

- Find the "right" problem formulation
- Use the right solver for the problem

Outline

Optimal Control – Problem Motivation

Car with gas pedal $u(t)$ as control input:
How to drive from A to B on a given time with minimum fuel consumption?

- Infinite dimensional decision variable $u(t)$.
- Cost function $\int_0^{t_f} \dot{m}_f(t)dt$
- Constraints:
  - Model of the car (the vehicle motion equation)

$$\begin{array}{rcll} m_v \frac{d}{dt} v(t) & = & F_t(v(t), u(t)) & -(F_a(v(t)) + F_r(v(t)) + F_g(x(t))) \\ \frac{d}{dt} x(t) & = & v(t) & \\ \dot{m}_f & = & f(v(t), u(t)) & \end{array}$$

  - Starting point $x(0) = A$
  - End point $x(t_f) = B$
  - Speed limits $v(t) \leq g(x(t))$
  - Limited control action $0 \leq u(t) \leq 1$
- In general difficult (impossible) problem to solve analytically.

General problem formulation

- Performance index

$$J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t)dt$$

- System model (constraints)

$$\frac{d}{dt}x = f(x(t), u(t), t), x(t_a) = x_a$$

- State and control constraints

$$u(t) \in U(t)$$

$$x(t) \in X(t)$$

Optimal Control – Historical Perspective

- Old subject
- Rich theory
  - Old theory from calculus of variations
  - Much theory and many methods were developed during 50's-70's
  - Theory and methods are still being actively developed
- Dynamic programming, Richard Bellman, 50's.
- A modern success story:
  –Model predictive control (MPC)
- Now a new interest for collocation methods:
  –A few during 1990's
  –Much interest 2000–

Outline

Dynamic programming – Problem Formulation

- Optimal control problem

$$\min J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t)dt$$

$$\begin{aligned} s.t.\ \frac{d}{dt}x &= f(x(t), u(t), t) \\ x(t_a) &= x_a \\ u(t) &\in U(t) \\ x(t) &\in X(t) \end{aligned}$$

- $x(t)$, $u(t)$ functions on $t \in [t_a, t_b]$
- Search an approximation to the solution by discretizing
  - the state space $x(t)$
  - and maybe the control signal $u(t)$

  in both amplitude and time.
- The result is a combinatorial (network) problem

Dynamic Programming (DP) – Problem Formulation

- Find the optimal control sequence $\pi^0(x_0) = \{u_0, u_1, \ldots, u_{N-1}\}$ minimizing:

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, w_k)$$

- subject to:

$$\begin{aligned} x_{k+1} &= f_k(x_k, u_k, w_k) \\ x_0 &= x(t=0) \\ x_k &\in X_k \\ u_k &\in U_k \end{aligned}$$

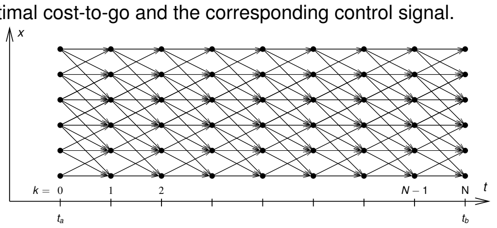
- Disturbance w_k
- Stochastic vs deterministic DP

Deterministic Dynamic Programming – Basic algorithm

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$

$$x_{k+1} = f_k(x_k, u_k)$$

Algorithm idea:
Start at the end and proceed backward in time to evaluate the optimal cost-to-go and the corresponding control signal.



Deterministic Dynamic Programming – Basic algorithm

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$

$$x_{k+1} = f_k(x_k, u_k)$$

Algorithm:

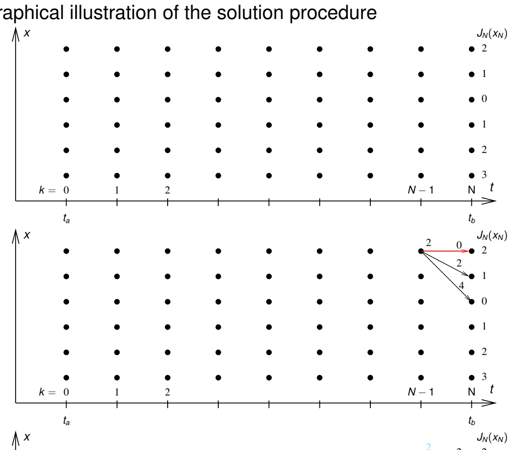
1. Set $k = N$, and assign final cost $J_N(x_N) = g_N(x_N)$
2. Set $k = k - 1$
3. For all points in the state-space grid, find the optimal cost to go

$$J_k(x_k) = \min_{u_k \in U_k(x_k)} g_k(x_k, u_k) + J_{k+1}(f_k(x_k, u_k))$$

4. If $k = 0$ then return solution
5. Go to step 2

Deterministic Dynamic Programming – Basic Algorithm

Graphical illustration of the solution procedure



Arc Cost Calculations

There are two ways for calculating the arc costs

- Calculate the exact control signal and cost for each arc.
  –Quasi-static approach
- Make a grid over the control signal and interpolate the cost for each arc.
  –Forward calculation approach

Matlab implementation – it is important to utilize matrix calculations

- Calculate the whole bundle of arcs in one step
- Add boundary and constraint checks

Pros and Cons with Dynamic Programming

Pros

- Globally optimal, for all initial conditions
- Can handle nonlinearities and constraints
- Time complexity grows linearly with horizon
- Use output and solution as reference for comparison

Cons

- Non causal
- Time complexity grows “exponentially” with number of states
- Only open loop scheme

Calculation Example

- Problem 200s with discretization $\Delta t = 1$s.
- Control signal discretized with 10 points.
- Statespace discretized with 1000 points.
- One evaluation of the model takes 1μs
- Solution time:
  - Brute force:
    Evaluate all possible combinations of control sequences.
    Number of evaluations, 10^{200} gives $\approx 3 \cdot 10^{186}$ years.
  - Dynamic programming:
    Number of evaluations: $200 \cdot 10 \cdot 1000$ gives 2 s.

This example comes from ETH slides

The Provided Tools for Hand-in Assignment 2

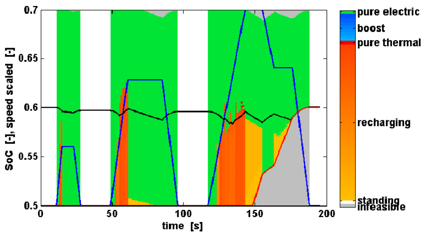
Task:
Investigate optimal control of one parallel and one series hybrid configuration in different driving profiles.

- Some Matlab-functions provided
  - Skeleton file for defining the problems
  - 2 DDP solvers, 1-dim and 2-dim.
  - 2 skeleton files for calculating the arc costs for parallel and serial hybrids

Outline

Parallel Hybrid Example

- Fuel-optimal torque split factor $u(SOC, t) = \frac{T_{e-motor}}{T_{gearbox}}$
- ECE cycle
- Constraints $SOC(t = t_f) \geq 0.6$, $SOC \in [0.5, 0.7]$



Parallel Hybrid Example

- Fuel-optimal torque split factor $u(SOC, t) = \frac{T_{e-motor}}{T_{gearbox}}$
- NEDC cycle
- Constraints $SOC(t = t_f) = 0.6$, $SOC \in [0.5, 0.7]$