

Vehicle Propulsion Systems

Lecture 5

Deterministic Dynamic Programming and Some Examples

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Outline

Repetition

"Traditional" Optimization

Problem motivation
Different Classes of Problems

Optimal Control

Problem Motivation

Deterministic Dynamic Programming

Problem setup and basic solution idea
Cost Calculation – Two Implementation Alternatives
The Provided Tools

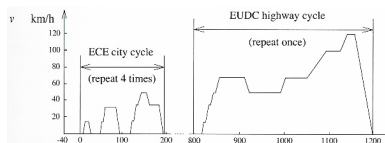
Case Studies

Energy Management of a Parallel Hybrid

1/31

2/31

Energy consumption for cycles



Numerical values for MVEG-95, ECE, EUDC

$$\text{air drag} = \frac{1}{x_{\text{tot}}} \sum_{i \in \text{trac}} \bar{v}_i^3 h = \{319, 82.9, 455\}$$

$$\text{rolling resistance} = \frac{1}{x_{\text{tot}}} \sum_{i \in \text{trac}} \bar{v}_i h = \{.856, 0.81, 0.88\}$$

$$\text{kinetic energy} = \frac{1}{x_{\text{tot}}} \sum_{i \in \text{trac}} \bar{a}_i \bar{v}_i h = \{0.101, 0.126, 0.086\}$$

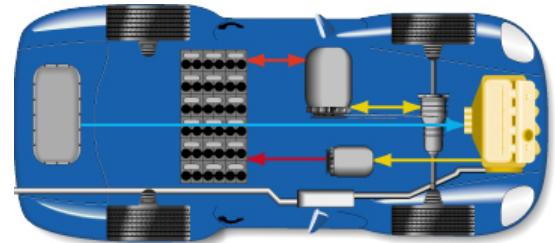
$$\bar{E}_{\text{MVEG-95}} \approx A_f c_d 1.9 \cdot 10^4 + m_v c_r 8.4 \cdot 10^2 + m_v 10 \quad \text{kJ/100km}$$

3/31

4/31

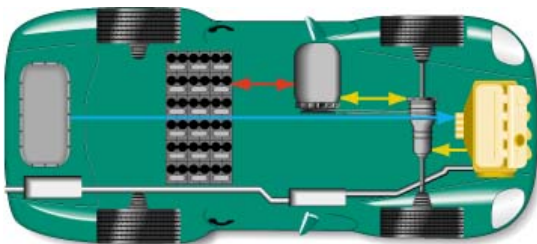
Hybrid Electrical Vehicles – Serial

- ▶ Two paths working in parallel
- ▶ Decoupled through the battery



Hybrid Electrical Vehicles – Parallel

- ▶ Two parallel energy paths



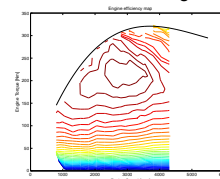
5/31

6/31

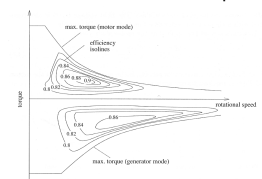
Component modeling

- ▶ Model energy (power) transfer and losses
- ▶ Using maps $\eta = f(T, \omega)$

Combustion engine map



Electric motor map



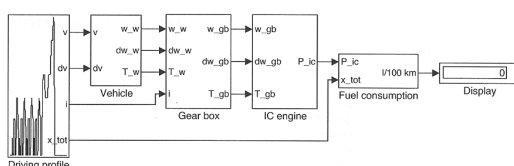
- ▶ Using parameterized (scalable) models
–Willans approach

5/31

6/31

Model implemented in QSS

Conventional powertrain



Efficient computations are important
–For example if we want to do optimization and sensitivity studies.

7/31

8/31

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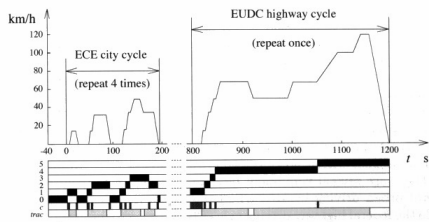
Case Studies

Energy Management of a Parallel Hybrid

Problem motivation

What gear ratios give the lowest fuel consumption for a given driving cycle?

– Problem presented in appendix 8.1



Problem characteristics

- ▶ Countable number of free variables, $i_{g,j}, j \in [1, 5]$
- ▶ A “computable” cost, $m_f(\dots)$
- ▶ A “computable” set of constraints, model and cycle
- ▶ The formulated problem

$$\begin{aligned} \min_{i_{g,j}, j \in [1,5]} \quad & m_f(i_{g,1}, i_{g,2}, i_{g,3}, i_{g,4}, i_{g,5}) \\ \text{s.t.} \quad & \text{model and cycle is fulfilled} \end{aligned}$$

9/31

Optimization – Linear Programming

- ▶ Linear problem

$$\begin{aligned} \min_x \quad & c^T x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

- ▶ Convex problem
- ▶ Much analyzed: existence, uniqueness, sensitivity
- ▶ Many algorithms: Simplex the most famous

- ▶ About the word *Programming*

– The solution to a problem was called a program

10/31

Optimization – Non-Linear Programming

- ▶ Non-linear problem

$$\begin{aligned} \min_x \quad & f(x) \\ \text{s.t.} \quad & g(x) = 0 \\ & x \geq 0 \end{aligned}$$

- ▶ For convex problems
 - Much analyzed: existence, uniqueness, sensitivity
 - Many algorithms
- ▶ For non-convex problems
 - Some special problems have solutions
 - Local optimum is not necessarily a global optimum

11/31

Mixed Integer and Combinatorial Optimization

- ▶ Problem

$$\begin{aligned} \min_{x,y} \quad & f(x, y) \\ \text{s.t.} \quad & g(x, y) = 0 \\ & x \geq 0 \\ & y \in \mathbb{Z}^+ \end{aligned}$$

- ▶ Inherently non-convex y
 - Generally hard problems to solve.
- ▶ Much analyzed
 - Existence, uniqueness, sensitivity
 - Many types of problems
 - Many different algorithms

12/31

Some comments on problem solver

- ▶ Find the “right” problem formulation
- ▶ Use the right solver for the problem

13/31

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14/31

Optimal Control – Problem Motivation

Car with gas pedal $u(t)$ as control input:

How to drive from A to B on a given time with minimum fuel consumption?

- ▶ Infinite dimensional decision variable $u(t)$.
- ▶ Cost function $\int_0^t \dot{m}_f(t) dt$
- ▶ Constraints:
 - ▶ Model of the car (the vehicle motion equation)

$$\begin{aligned} m_v \frac{d}{dt} v(t) &= F_t(v(t), u(t)) - (F_a(v(t)) + F_r(v(t)) + F_g(x(t))) \\ \frac{d}{dt} x(t) &= v(t) \\ \dot{m}_f &= f(v(t), u(t)) \end{aligned}$$

- ▶ Starting point $x(0) = A$
- ▶ End point $x(t_f) = B$
- ▶ Speed limits $v(t) \leq g(x(t))$
- ▶ Limited control action $0 \leq u(t) \leq 1$
- ▶ Difficult (impossible) problem to solve analytically

15/31

General problem formulation

- ▶ Performance index

$$J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t) dt$$

- ▶ System model (constraints)

$$\frac{d}{dt} x = f(x(t), u(t), t), x(t_a) = x_a$$

- ▶ State and control constraints

$$u(t) \in U(t)$$

$$x(t) \in X(t)$$

16/31

- ▶ Old subject
- ▶ Rich theory
 - ▶ Old theory from calculus of variations
 - ▶ Much theory and many methods were developed during 50's-70's
 - ▶ Theory and methods are still being actively developed
- ▶ Dynamic programming, Richard Bellman, 50's.
- ▶ A modern success story:
 - Model predictive control (MPC)
- ▶ Now a new interest for collocation methods:
 - A few during 1990's
 - Much interest 2000–

17/31

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18/31

Dynamic programming – Problem Formulation

- ▶ Optimal control problem

$$\begin{aligned} \min J(u) &= \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t) dt \\ \text{s.t. } \frac{d}{dt}x &= f(x(t), u(t), t) \\ x(t_a) &= x_a \\ u(t) &\in U(t) \\ x(t) &\in X(t) \end{aligned}$$

- ▶ $x(t)$, $u(t)$ functions on $t \in [t_a, t_b]$
- ▶ Search an approximation to the solution by discretizing
 - ▶ the state space $x(t)$
 - ▶ and maybe the control signal $u(t)$
 in both amplitude and time.
- ▶ The result is a combinatorial (network) problem

19/31

Dynamic Programming (DP) – Problem Formulation

- ▶ Find the optimal control sequence $\pi^0(x_0) = \{u_0, u_1, \dots, u_{N-1}\}$ minimizing:

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, w_k)$$

- ▶ subject to:

$$\begin{aligned} x_{k+1} &= f_k(x_k, u_k, w_k) \\ x_0 &= x(t=0) \\ x_k &\in X_k \\ u_k &\in U_k \end{aligned}$$

- ▶ Disturbance w_k
- ▶ Stochastic vs Deterministic DP

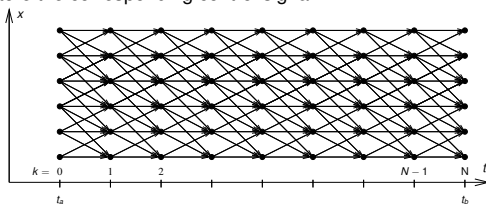
20/31

DDP – Basic Algorithm

$$\begin{aligned} J(x_0) &= g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k) \\ x_{k+1} &= f_k(x_k, u_k) \end{aligned}$$

Bellman's Theory and Algorithm:

- Start at the end and proceed backward in time
- Determine the optimal cost-to-go
- Store the corresponding control signal



21/31

DDP – Basic algorithm

$$\begin{aligned} J(x_0) &= g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k) \\ x_{k+1} &= f_k(x_k, u_k) \end{aligned}$$

Algorithm:

1. Set $k = N$, and assign final cost $J_N(x_N) = g_N(x_N)$
2. Set $k = k - 1$
3. For all points in the state-space grid, find the optimal cost to go

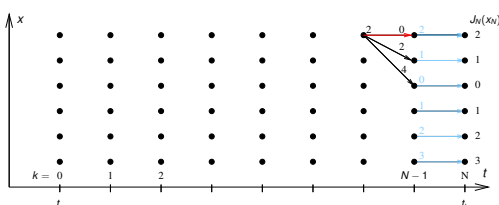
$$J_k(x_k) = \min_{u_k \in U_k(x_k)} g_k(x_k, u_k) + J_{k+1}(f_k(x_k, u_k))$$

4. If $k = 0$ then return solution
5. Go to step 2

22/31

Deterministic Dynamic Programming – Basic Algorithm

Graphical illustration of the solution procedure



23/31

Arc Cost Calculations

There are two ways for calculating the arc costs

- ▶ Calculate the exact control signal and cost for each arc.
 - Quasi-static approach
- ▶ Make a grid over the control signal and interpolate the cost for each arc.
 - Forward calculation approach

Matlab implementation – it is important to utilize matrix calculations

- ▶ Calculate the whole bundle of arcs in one step
- ▶ Add boundary and constraint checks

24/31

Pros

- Globally optimal, for all initial conditions
- Can handle nonlinearities and constraints
- Time complexity grows linearly with horizon
- Use output and solution as reference for comparison

Cons

- Non causal
- Time complexity grows “exponentially” with number of states

25/31

- Problem 200s with discretization $\Delta t = 1s$.
- Control signal discretized with 10 points.
- Statespace discretized with 1000 points.
- One evaluation of the model takes $1\mu s$
- Solution time:
 - Brute force:
Evaluate all possible combinations of control sequences.
Number of evaluations, 10^{200} gives $\approx 3 \cdot 10^{186}$ years.
 - Dynamic programming:
Number of evaluations: $200 \cdot 10 \cdot 1000$ gives 2 s.

(Example contributed by ETH)

26/31

The Provided Tools for Hand-in Assignment 2

Task:

Investigate optimal control of one parallel and one series hybrid configuration in different driving profiles.

- Some Matlab-functions provided
 - Skeleton file for defining the problems
 - 2 DDP solvers, 1-dim and 2-dim.
 - 2 skeleton files for calculating the arc costs for parallel and serial hybrids

27/31

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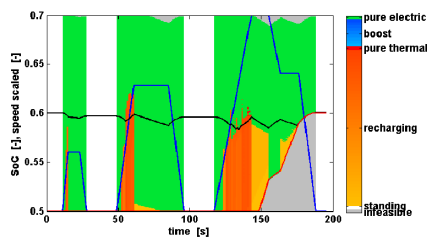
Case Studies

- Energy Management of a Parallel Hybrid

28/31

Parallel Hybrid Example

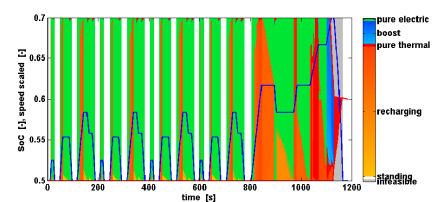
- Fuel-optimal torque split factor $u(SOC, t) = \frac{T_{g-motor}}{T_{gearbox}}$
- ECE cycle
- Constraints $SOC(t = t_f) \geq 0.6$, $SOC \in [0.5, 0.7]$



29/31

Parallel Hybrid Example

- Fuel-optimal torque split factor $u(SOC, t) = \frac{T_{g-motor}}{T_{gearbox}}$
- NEDC cycle
- Constraints $SOC(t = t_f) = 0.6$, $SOC \in [0.5, 0.7]$



30/31