

# AN OPTIMAL CONTROL BENCHMARK: TRANSIENT OPTIMIZATION OF A DIESEL-ELECTRIC POWERTRAIN

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## ABSTRACT

An optimal control benchmark is presented and discussed. The benchmark is optimal transient control of a nonlinear four state three control model of a diesel-electric powertrain and constructed in such a manner that it is available in several versions to be of interest for developers of optimal control tools at different levels of development. This includes with and without time as a parameter as well as with and without time varying constraints.

*Keywords:* Optimal Control, Diesel engine, Diesel-Electric, Nonlinear optimization

## NOMENCLATURE

Symbol	Description	Unit
$p$	Pressure	Pa
$T$	Temperature	K
$\omega$	Rotational speed	rad/s
$\dot{m}$	Massflow	kg/s
$P$	Power	W
$M$	Torque	Nm
$E$	Energy	J
$\Pi$	Pressure ratio	-
$V$	Volume	$m^3$
$R$	Gas Constant	J/(kg · K)
$u_f, u_{wg}, P_{gen}$	Control signals	mg/cycle, -, W
$J$	Inertia	$kg \cdot m^2$
$BSR$	Blade speed ratio	-
$\phi$	Fuel-air equivalence ratio	-
$\lambda_{min}$	Air-fuel smoke-limit	-

Table 1: Symbols used

## INTRODUCTION

In this paper a benchmark for optimal control tools is suggested and presented. The current state of computer technology has enabled a rise in the development of optimal control packages that can handle models of complex systems. However to evaluate the performance of the tools developers often have to rely on relatively small problems that do not reflect the purpose for which the tools were developed.

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Index	Description	Index	Description
$ice$	Engine	$GenSet$	Engine-Generator
$im$	Intake manifold	$em$	Exhaust manifold
$c$	Compressor	$ac$	After compressor
$t$	Turbine	$wg$	Wastegate
$f$	Fuel	$tc$	Turbocharger
$a$	Air	$e$	Exhaust
$gen$	Generator-electrical	$mech$	Generator-mechanical
$req$	Requested	$c, surge$	Compressor surge-limit

Table 2: Subscripts used

This paper presents a benchmark on which to evaluate developed optimal control tools. The benchmark problem is the optimization of the control of a diesel-electric powertrain from idle to a target output power and energy. The benchmark relies on the validated model of a diesel-electric powertrain described in [1]. The model is a nonlinear four state, three controls mean value engine model (MVEM), that consists of 10 submodels that describe the individual components of the powertrain. Due to the complex and nonlinear nature of the modeled system the resulting optimization problem is non-convex and the optimization tools can therefore only guarantee local optima. The model is continuously differentiable in the desired operating region and is implemented using only analytical expressions. The motivation for this is to enable the solvers to use higher order search methods in the optimization. It also makes the model suitable for automatic differentiation (AD), enabling developers to also evaluate

AD routines versus computing gradients and Hessians using finite differences.

In the paper the solutions to the problems using two different solvers, the ACADO Toolkit, see [2], TOMLAB/PROPT, see [3], is presented and discussed. The model and the resulting optimal trajectories, as well as the corresponding initial guesses, are available for research community. Two types of problems are considered, time and fuel minimization. To make the benchmark problem suitable for optimal control tools at different stages of development the problems are solved both with duration as a parameter to be optimized as well as for a fixed duration and also with and without path constraints.

## CONTRIBUTIONS

The contribution of this paper is the formulation and solution of an optimal control problem to serve as a benchmark on which to evaluate optimal control. The intention of the benchmark is to provide the research community with a relevant problem of reasonable complexity on which to benchmark optimal control tools. The benchmark is provided together with a simultaneously developed model, both available for download. To ensure that the benchmark is relevant for tools at different stages of development the problem is provided both with and without path constraints as well as with and without time as a parameter.

## MODEL

The model used can be downloaded from [4] and is described in detail as *MVEM*<sub>2</sub> in [1], and provided either on its own, in the **LiU-D-El**-package, or together with the benchmark in the **LiU-D-El+Benchmark**-package. The modeled diesel-electric powertrain consists of a 6-cylinder diesel engine with a fixed-geometry turbine and a wastegate for boost control, with a generator mounted on the output shaft. The states of the MVEM are engine speed,  $\omega_{ice}$ , inlet manifold pressure,  $p_{im}$ , exhaust manifold pressure,  $p_{em}$ , and turbocharger speed,  $\omega_{tc}$ . The controls are injected fuel mass,  $u_f$ , wastegate position,  $u_{wg}$ , and generator power,  $P_{gen}$ . The engine model consists of two control volumes, intake and exhaust manifold, and four restrictions, compressor, engine, turbine, and wastegate. The control volumes are modeled with the standard isothermal model, us-

ing the ideal gas law and mass conservation. The engine and turbocharger speeds are modeled using Newton's second law. The governing differential equations of the MVEM are:

$$\frac{d\omega_{ice}}{dt} = \frac{P_{ice} - P_{mech}}{\omega_{ice}J_{GenSet}} \quad (1)$$

$$\frac{dp_{im}}{dt} = \frac{R_a T_{im}}{V_{im}} (\dot{m}_c - \dot{m}_{ac}) \quad (2)$$

$$\frac{dp_{em}}{dt} = \frac{R_e T_{em}}{V_{em}} (\dot{m}_{ac} + \dot{m}_f - \dot{m}_t - \dot{m}_{wg}) \quad (3)$$

$$\frac{d\omega_{tc}}{dt} = \frac{P_t \eta_{tm} - P_c}{\omega_{tc}J_{tc}} \quad (4)$$

The MVEM is extended with two summation states to keep track on produced and consumed energy. The summation states are defined as:

$$\frac{dm_f}{dt} = \dot{m}_f \quad (5)$$

$$\frac{dE_{gen}}{dt} = P_{gen} \quad (6)$$

## PROBLEM FORMULATION

The proposed benchmark problem is the same problem as is studied in [5, 6]. The problem is that the GenSet is at idle when the operator requests a step in output power,  $P_{gen}$ , that should be met either as fuel efficient or time efficient as possible. The requested power is also augmented with an energy requirement,  $E_{gen}$ , that has to be produced before the GenSet reaches stationary conditions. This problem is mathematically expressed as:

$$\begin{aligned} \min_{u(t)} \quad & \int_0^T \dot{m}_f(x(t), u(t)) dt \quad \text{or} \quad \min_{u(t)} T \\ \text{s.t.} \quad & \dot{x}(t) = f(x(t), u(t)) \\ & (x(t), u(t)) \in \Omega(t) \end{aligned} \quad (7)$$

where  $x$  is the state vector of the MVEM,  $\dot{x}$  is the state equations (1)-(4) together with the summation states in (5)-(6), and  $u = [u_f, u_{wg}, P_{gen}]$ .

The constraints of the optimization problems,  $(x(t), u(t)) \in \Omega(t)$ , can be divided into two categories, time independent and time varying constraints. The first category, time independent constraints, are bounds on states and controls as well as

initial and final conditions expressed as:

$$\begin{aligned}
 x(0) &= x_0, & \dot{x}(T) &= 0 \\
 u_{min} &\leq u(t) \leq u_{max}, & x_{min} &\leq x(t) \leq x_{max} \\
 0 &\leq P_{gen}(t) \leq 100 \text{ kW}, & P_{gen}(T) &= 100 \text{ kW} \\
 E_{gen}(T) &\geq 100 \text{ kJ}
 \end{aligned} \tag{8}$$

The time varying constraints are constraints imposed by the components, such as maximum power of the engine, surge-limit of the compressor, blade speed ratio-limit of the turbine, as well as environmental constraints, i.e. an upper limit on  $\phi$  set by the smoke-limiter:

$$\begin{aligned}
 P_{ice}(x(t), u(t)) &\leq P_{ice,max}(x(t)) \\
 \Pi_c &\leq \Pi_{c,surge} \\
 BSR_{min} &\leq BSR(x(t), u(t)) \leq BSR_{max} \\
 0 &\leq \phi(x(t), u(t)) \leq \frac{1}{\lambda_{min}}
 \end{aligned} \tag{9}$$

To be relevant for software developers at different stages of development the benchmark problems defined in (7)-(9) are also available as a minimum fuel problem with fixed end time, as well as without the time varying constraints in (9).

## SOLUTION ACCURACY

To ensure that the solutions are at least good local minima both benchmark problems,  $\min T$  and  $\min m_f$ , are solved using PROPT and two different initial guesses, see Fig. 2-right. The first initial guess is a hard acceleration with  $\phi = \frac{1}{\lambda_{min}}$  from idle followed by a step in load power to  $P_{gen} = 100 \text{ kW}$ , and the second one is the GenSet at idle. Both initial guesses produce the same solution, although they are very different, indicating that the solutions are at least a good local minima. All solutions shown are with 125 control intervals/collocation points. In the following the initial guess from idle to 100kW is used.

## WITH TIME VARYING CONSTRAINTS

The benchmark problems defined by (7)-(9) are solved using ACADO and PROPT, and the solutions from the two solvers are shown and compared in Fig. 1, where  $\omega_{ice/tc}$  is engine speed and turbocharger speed,  $p_{im/em}$  intake and exhaust manifold

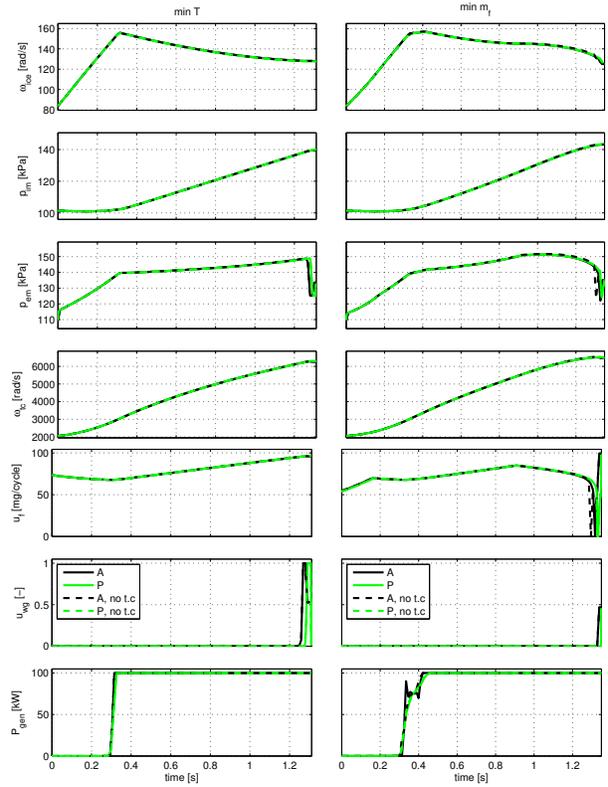


Figure 1: Minimum time (left) and minimum fuel (right) solutions to the benchmark problem using PROPT (P) and ACADO (A) with and without (no t.c) time varying constraints.

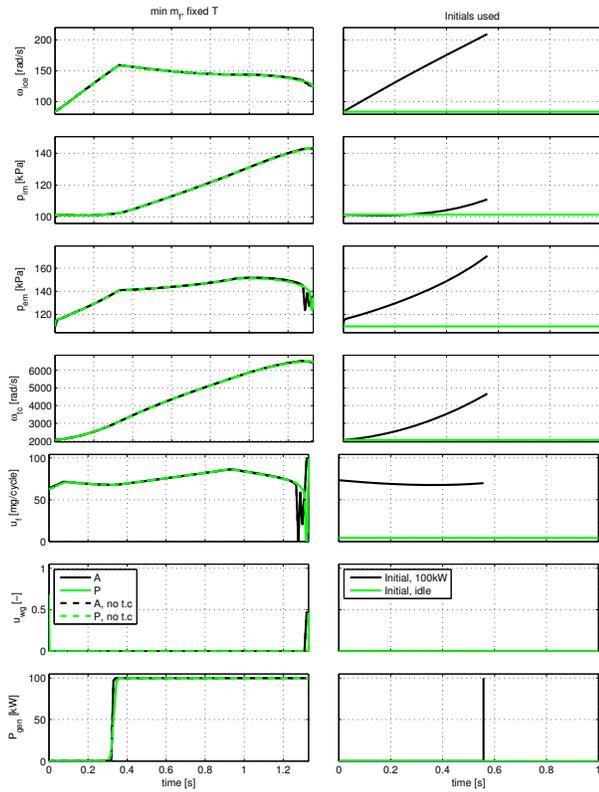


Figure 2: The fixed time solutions using PROPT (P) and ACADO (A) (left) with and without (no t.c) time varying constraints, as well as the different initial guesses used (right).

pressure,  $u_f/u_{wg}/P_{gen}$  are the controls, i.e. injected fuel mass per cycle, wastegate position, and output power from the generator. The  $\min m_f$  problem is also solved with fixed end time,  $T$ . For this a duration between the time optimal and fuel optimal durations is selected,  $T = 1.33$ . The solution with fixed end time is shown in Fig. 2-left.

Both solvers produce qualitatively the same solutions, there are however some differences owing to discretization technique employed as well as solution method. The resulting consumptions are shown in Table. 3. For further comparison all three solutions using PROPT are shown in torque-engine speed domain in Fig. 3.

Looking at Fig. 1 the trajectories for  $\min m_f$  and  $\min T$  are a bit different. For  $\min T$   $u_f$  follows the smoke-limit, i.e.  $\phi = \frac{1}{\lambda_{min}}$ , during the entire transient, whereas for  $\min m_f$  it is only smoke-limited  $0.17 \leq t \leq 0.91$  and  $t = T$ . During the initial acceleration engine efficiency is instead maximized, clearly seen in Fig. 3. The  $\min T$  solutions apply

Criteria	Tool	$m_f$	$T$
$\min m_f$	P	6.5917398e-03	1.3498815e+00
	A	6.5977572e-03	1.3497437e+00
$\min T$	P	6.6392313e-03	1.3103439e+00
	A	6.6466370e-03	1.3110366e+00
$\min m_f$ , fixed T	P	6.5933240e-03	1.3300000e+00
	A	6.6005188e-03	1.3300000e+00

Table 3: Time and fuel consumption to the benchmark problems using both PROPT (P) and ACADO (A).

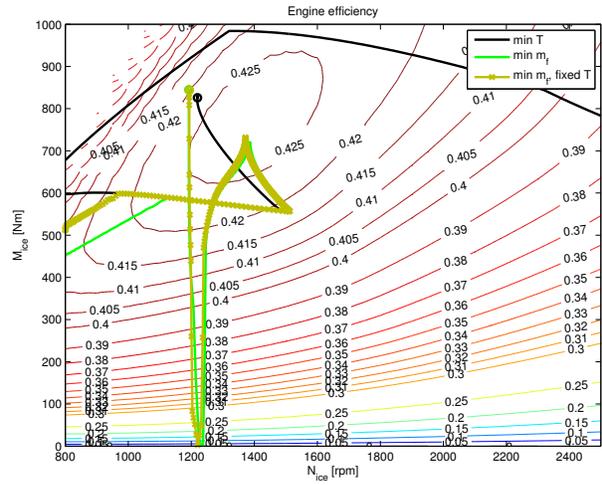


Figure 3: Torque-engine speed trajectories for the three benchmark problems.

a step in  $P_{gen}$  from  $0 \rightarrow 100$  kW whereas the  $\min m_f$   $P_{gen}$  actuation is a slightly later and not in a step from  $0 \rightarrow 100$  kW. At the end of the transient the  $\min m_f$  use both  $u_f$  and  $u_{wg}$  to bring the states to stationarity whereas  $\min T$  only uses  $u_{wg}$ . Noteworthy is that none of the solutions end in the peak efficiency operating point, neither of the GenSet nor of the ICE.

The  $\min m_f$ , fixed time, solutions are as expected a mix between the  $\min T/m_f$  solutions. The  $P_{gen}$  actuation follows that of  $\min T$  but  $u_f$  and  $u_{wg}$  are more similar to  $\min m_f$ .  $u_f$  during the initial acceleration does however not follow the maximum efficiency trajectory but instead follows a trajectory between this and the smoke-limit, see Fig.1.

## WITHOUT TIME VARYING CONSTRAINTS

In the problems solved in Section the only time varying constraint that is active is the smoke-limiter,

Criteria	Tool	$m_f$	$T$
$\min m_f$	P	6.5916926e-03	1.3497981e+00
	A	6.5985144e-03	1.3493627e+00
$\min T$	P	6.6394831e-03	1.3103321e+00
	A	6.6468526e-03	1.3104947e+00
$\min m_f, \text{ fixed } T$	P	6.5930972e-03	1.3300000e+00
	A	6.6000832e-03	1.3300000e+00

Table 4: Time and fuel consumption to the benchmark problems without time varying constraints using both PROPT (P) and ACADO (A).

i.e. the constraint on  $\phi$ . The others can therefore be removed without affecting the solution. For the problem without (9) to be relevant, i.e. the problem defined by (7)-(8), the smoke-limiter needs to be included. To achieve this without state-dependent time varying constraints the model is reformulated so that  $\phi$  is a control signal and  $u_f$  calculated from it. Since  $\phi = \frac{\dot{m}_f}{\dot{m}_{ac}}(A/F)_s$  and  $\dot{m}_f = \frac{10^{-6}}{4\pi}u_f\omega_{ice}n_{cyl}$  the model can be reformulated to:

$$\dot{m}_f = \frac{\phi \dot{m}_{ac}}{(A/F)_s} \quad (10)$$

$$u_f = \frac{4\pi}{10^{-6}} \frac{\dot{m}_f}{\omega_{ice}n_{cyl}} \quad (11)$$

and with  $\phi$  replacing  $u_f$  as control signal all the time varying constraints are removed. The solutions to the problem without time varying constraints follow, as expected, the same discussion as with time varying constraints. The results are also shown in Fig. 1-2 but the trajectories end up on top of each other. The fuel and time consumptions are shown in Table 4. The reformulation leads to slightly different numerical values but the difference is negligible.

## CONCLUSION

In this paper an optimal control benchmark is suggested and presented. The benchmark concerns transient optimization of a diesel-electric powertrain, from idle to a target power and energy. The benchmark makes use of a freely available four state-three control nonlinear model of a diesel-electric powertrain. Both the model and the initial guesses used are available for download in the **LiU-D-El+Benchmark**-package from [4]. The benchmark is available in several versions, both with and without time varying constraints, as well as with and without time as a parameter.

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